Mathematical Modeling in Modelica: The Art of Compressing Reality

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DLR
Institute of System Dynamics and Control

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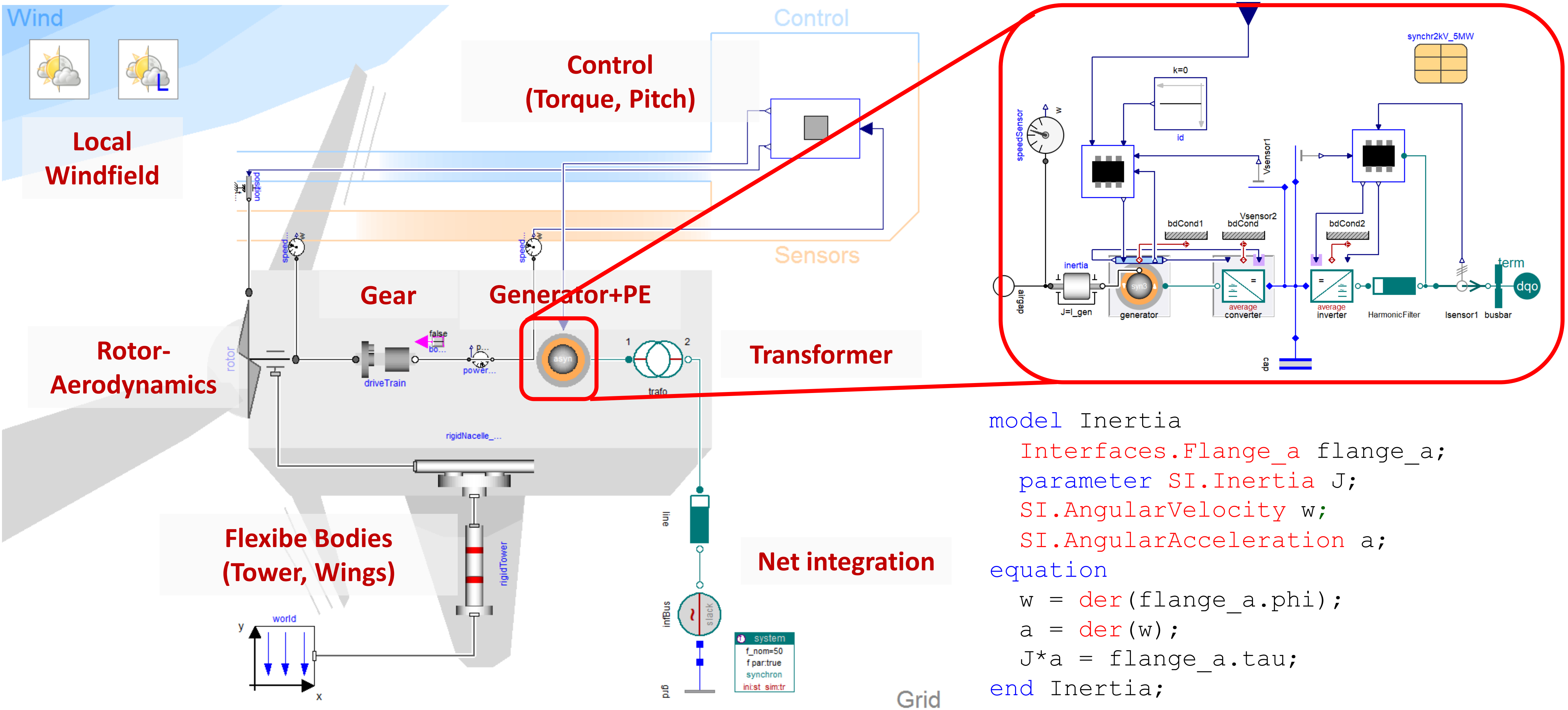
Equation-Based, Object-Oriented Modeling




```
model Inertia
  Interfaces.Flange_a flange_a;
  parameter SI.Inertia J;
  SI.AngularVelocity w;
  SI.AngularAcceleration a;
equation
  w = der(flange_a.phi);
  a = der(w);
  J*a = flange_a.tau;
end Inertia;
```

Basic Model Evaluation / Simulation

Multi-Domain Total System Composition

Object-Oriented System Composition

Idealized Component Classes

Fundamental Laws

Code Generation

Application-oriented part in imperative form

- Customer Deployment, Digital Twins
- Collaborative Design Processes, MBSE
- Local Design Optimization and Control
- Basic Model Evaluation / Simulation

Knowledge-oriented part in declarative form

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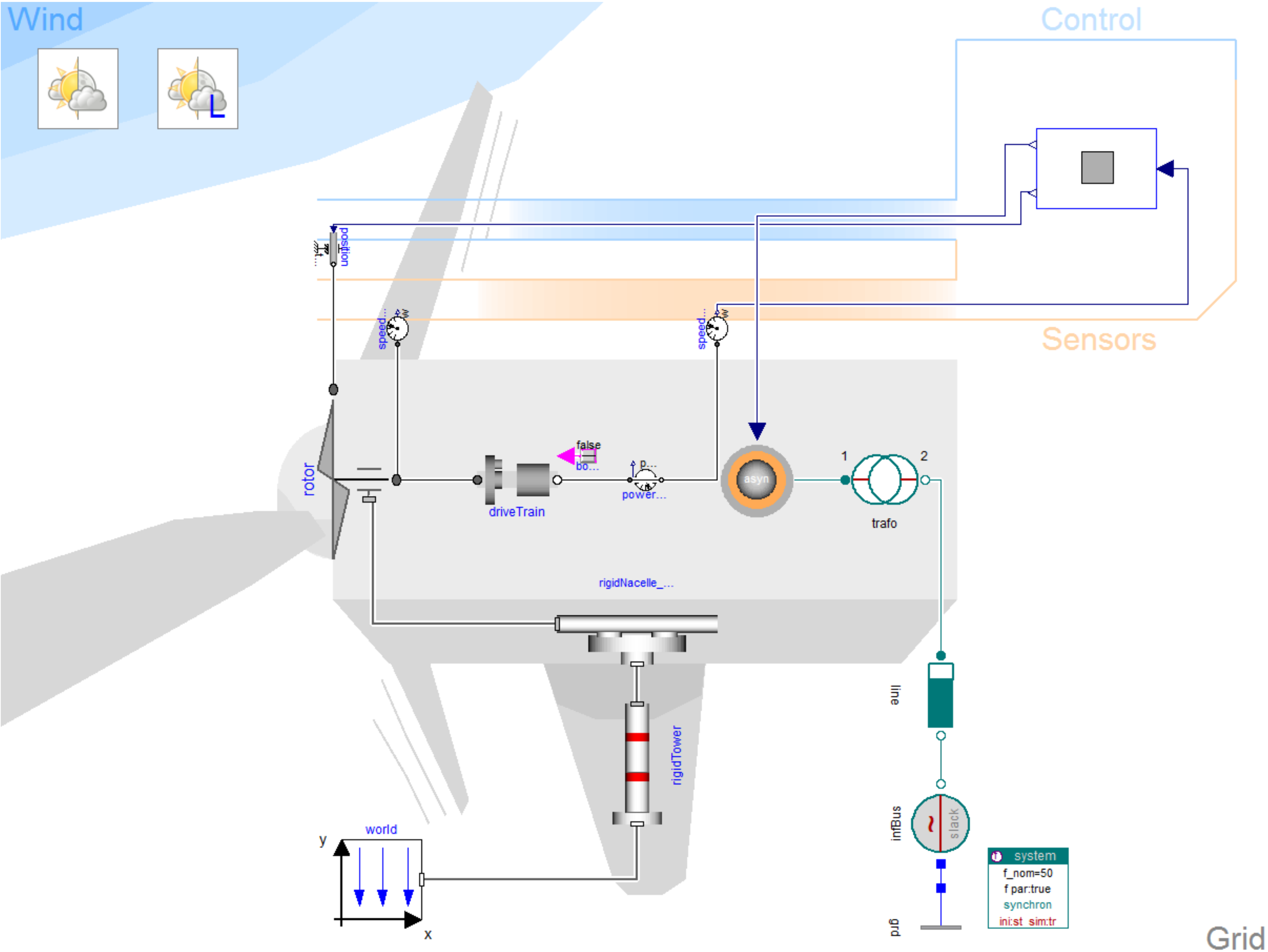
Wind
Control
Sensors
position
speed...
speed...
false
bo...
p...
power...
asyn
1
2
trafo
rotor
driveTrain
rigidNacelle_...
rigidTower
line
infBus
slack
grid
world
y
x
system
f_nom=50
f par:true
synchron
ini:st sim:tr
Grid

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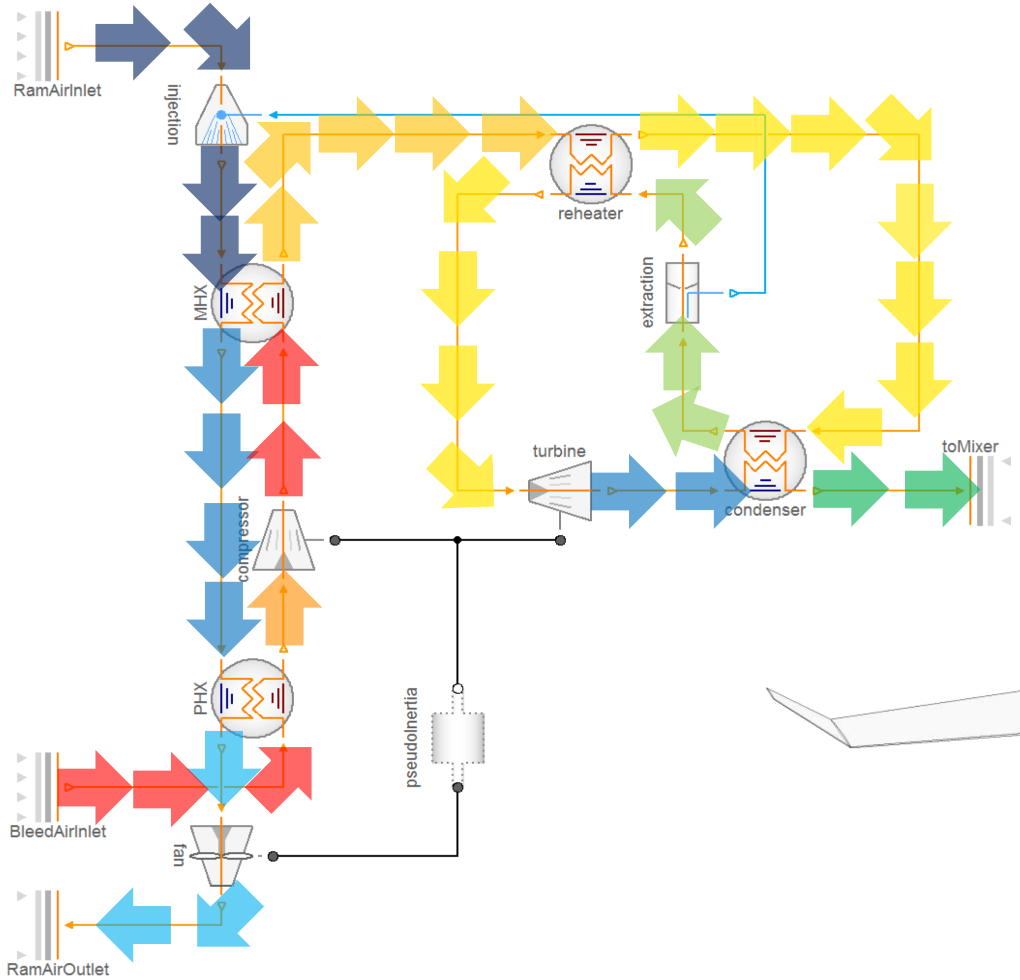
Simulation Error

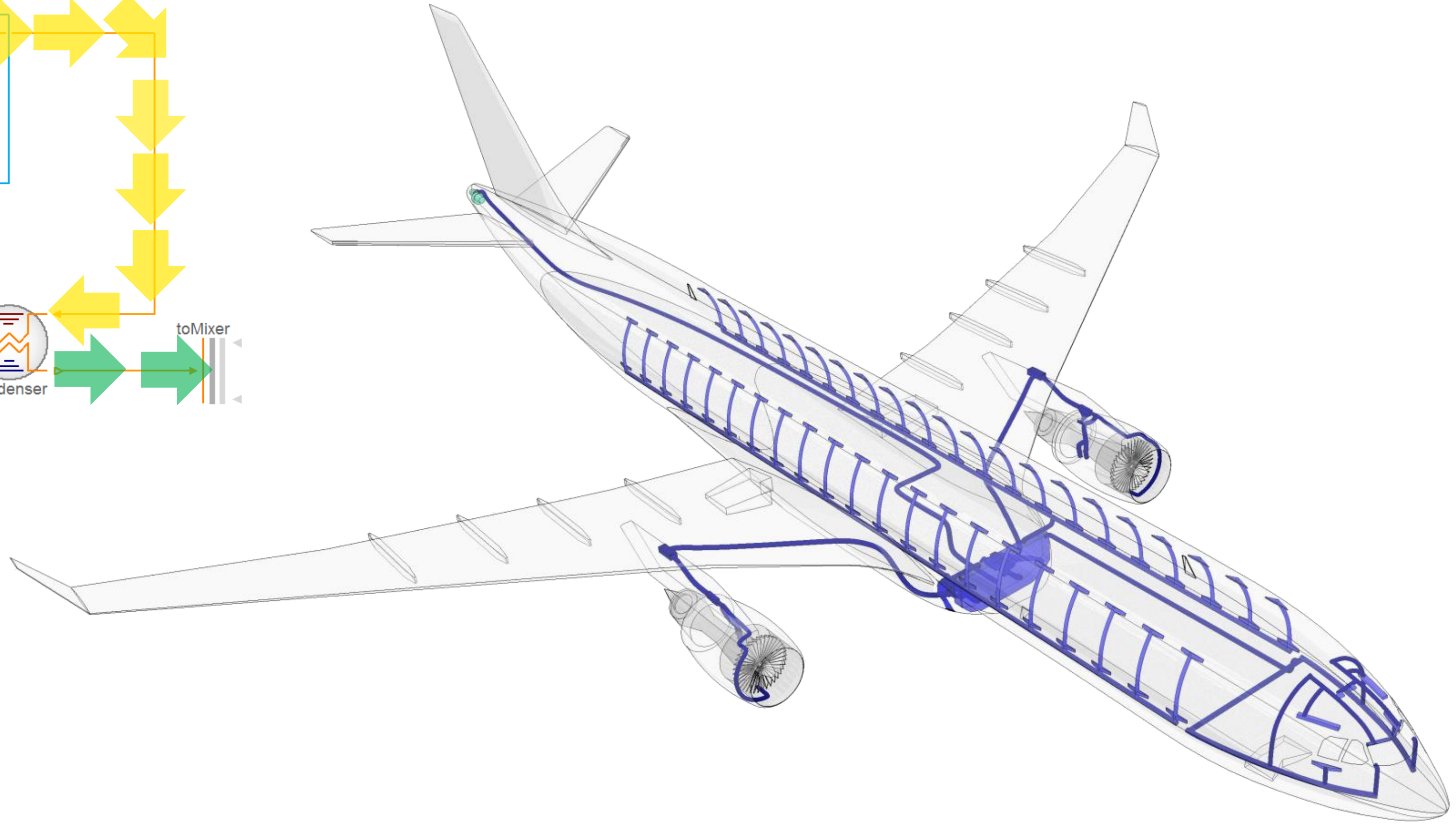
A fatal exception ocurred at 027:C8127 by the non-linear equation system solver.
Here is a cryptic error code that is of absolutely no use: 420.
Simulation has been stopped to prevent damage from your virtual universe.

*press any key to acknowledge defeat
*press Ctrl+Alt+Del if you think that this is any better
*by the way, we deleted your hard-drive

Press any key to continue _

Motivation Example: An Air Cycle





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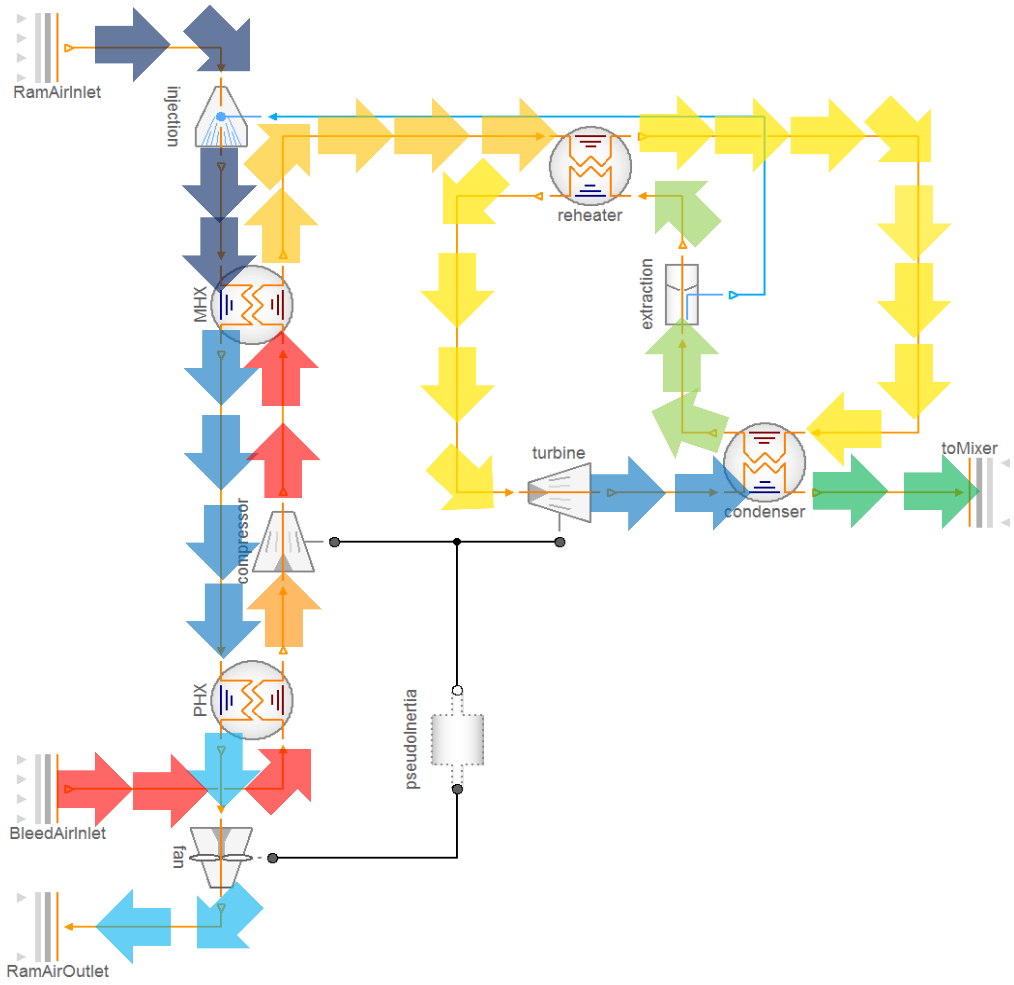

- This leads to a system with more than **200 non-linear equations** (with more than 40 iteration variables)

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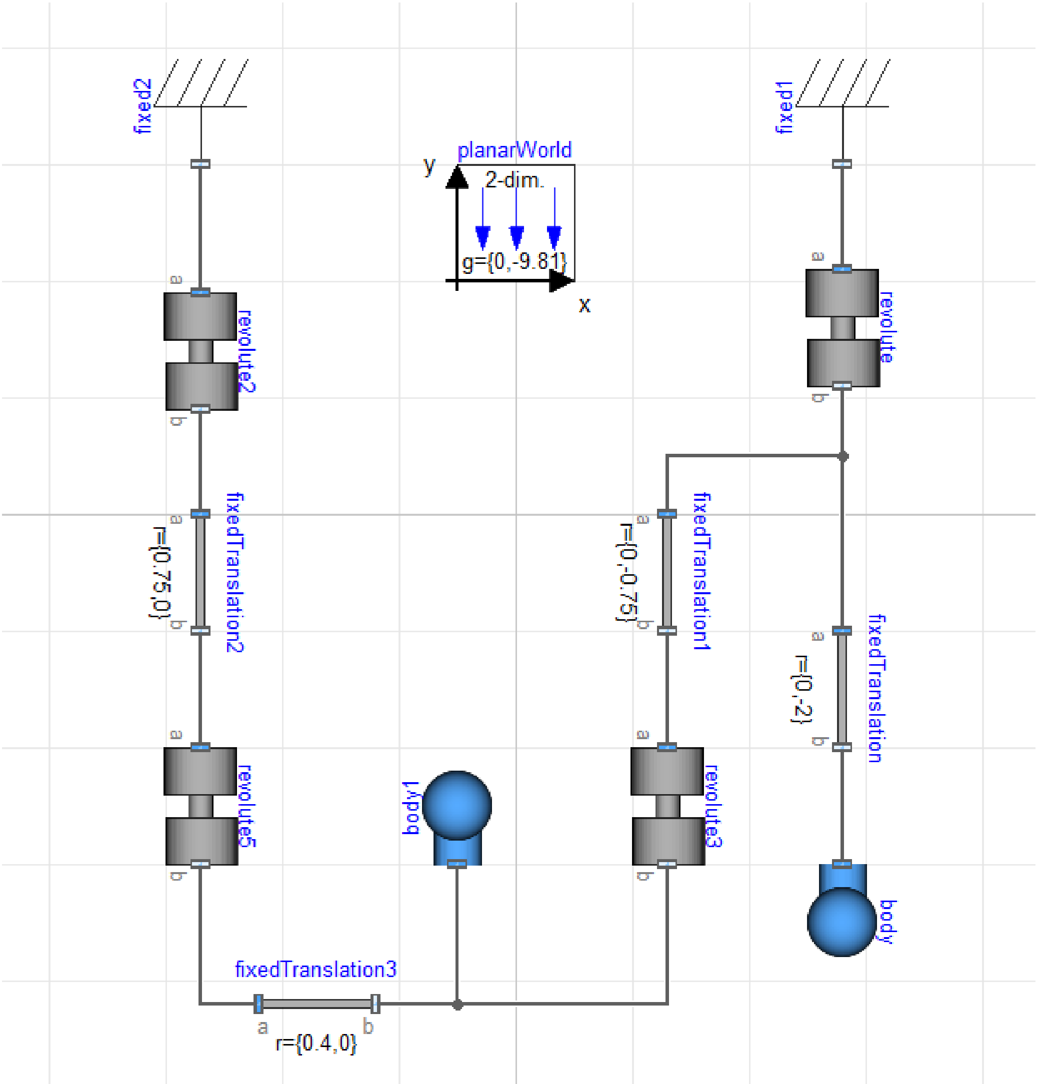
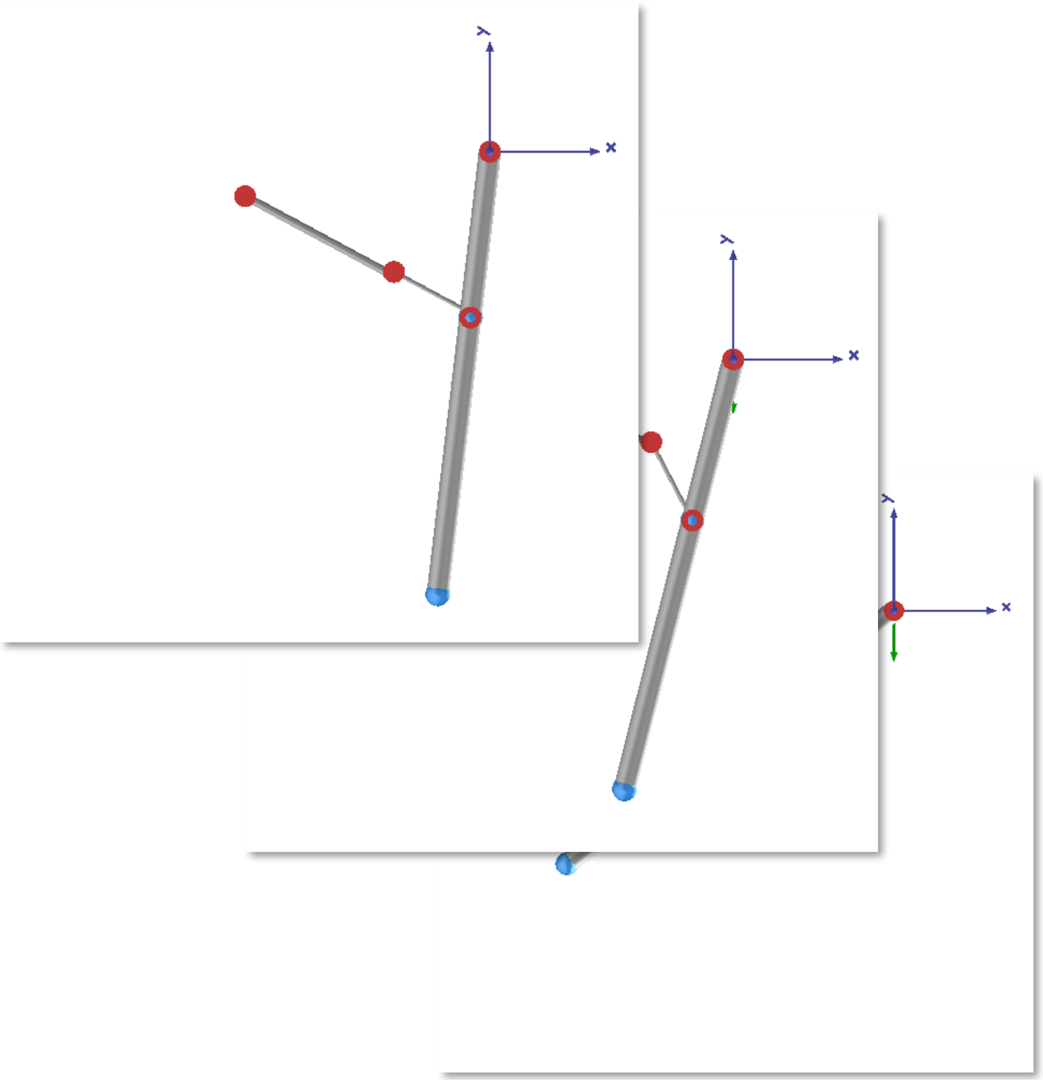
Simulation does not always work





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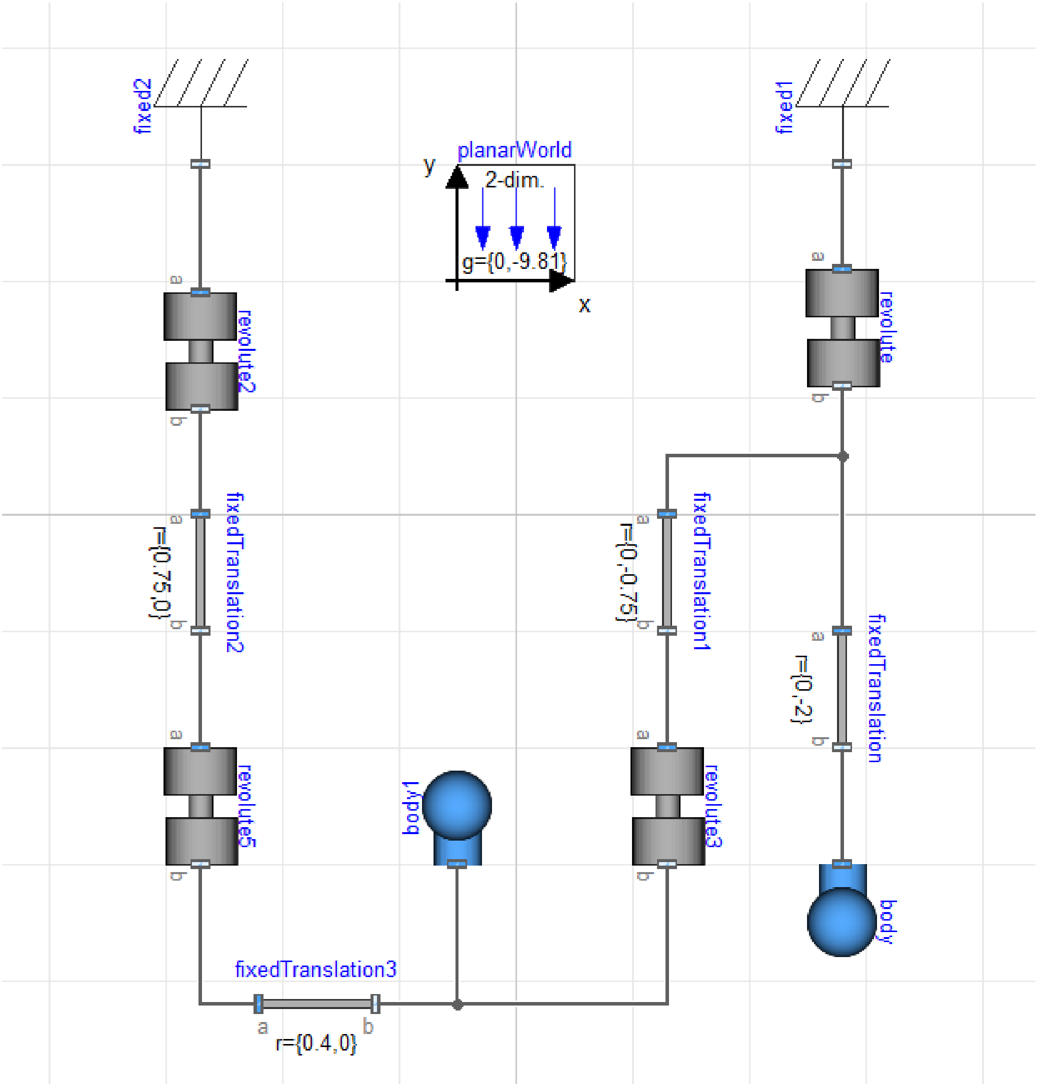

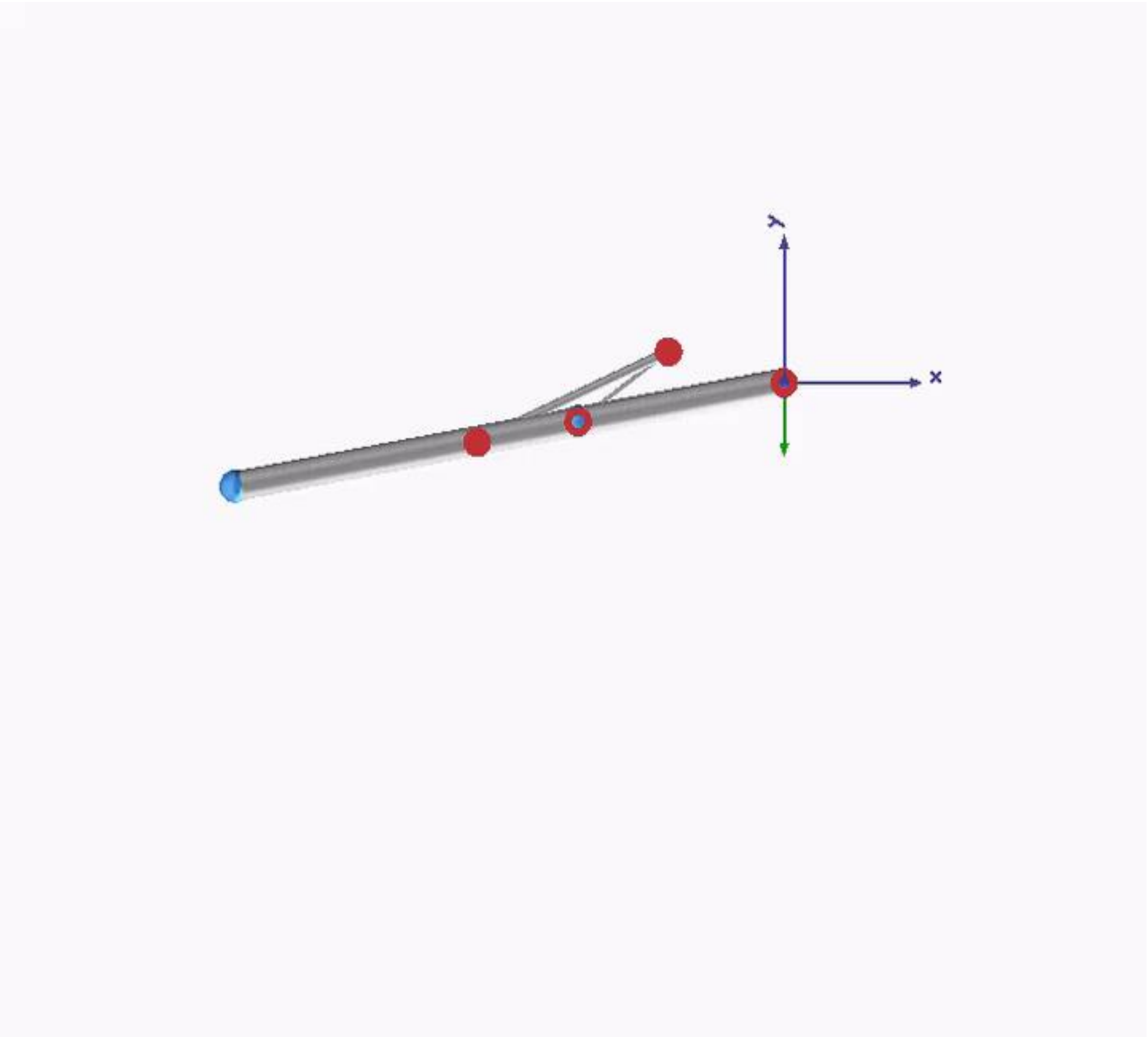

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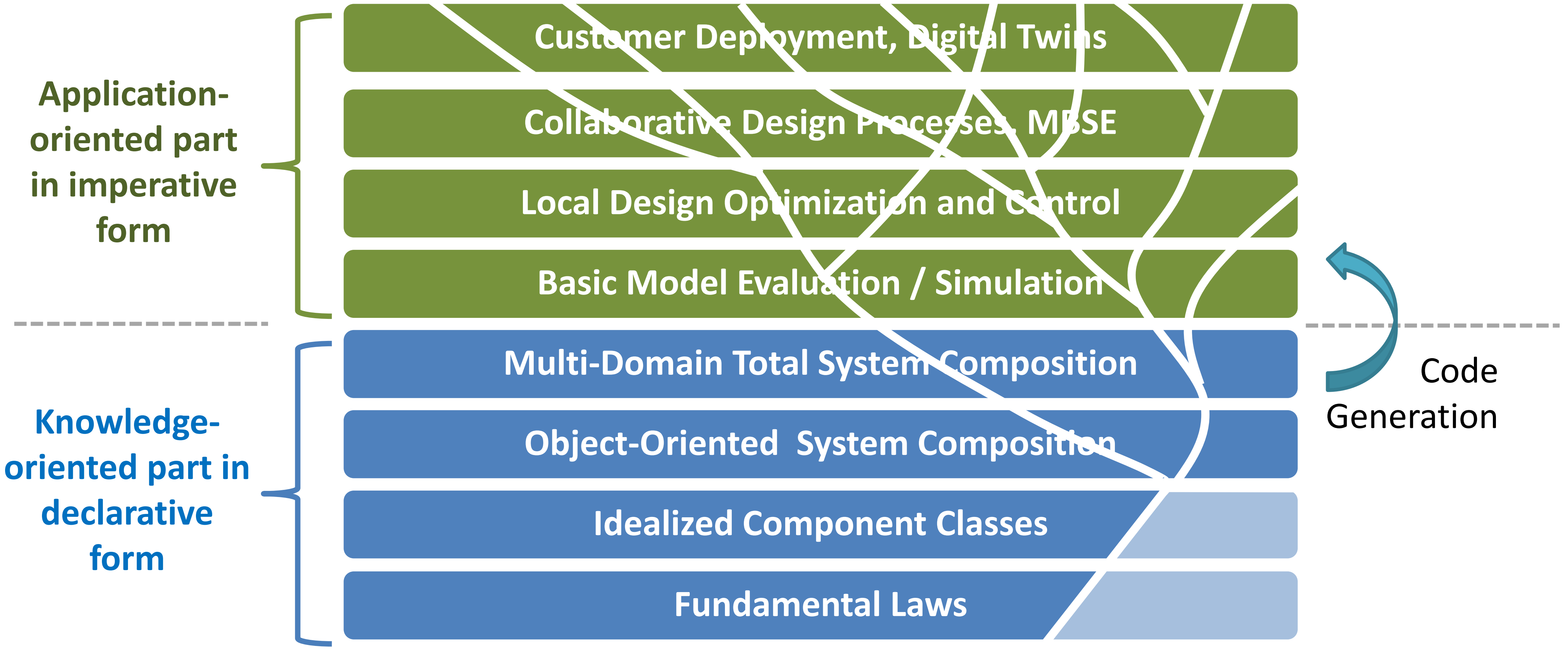
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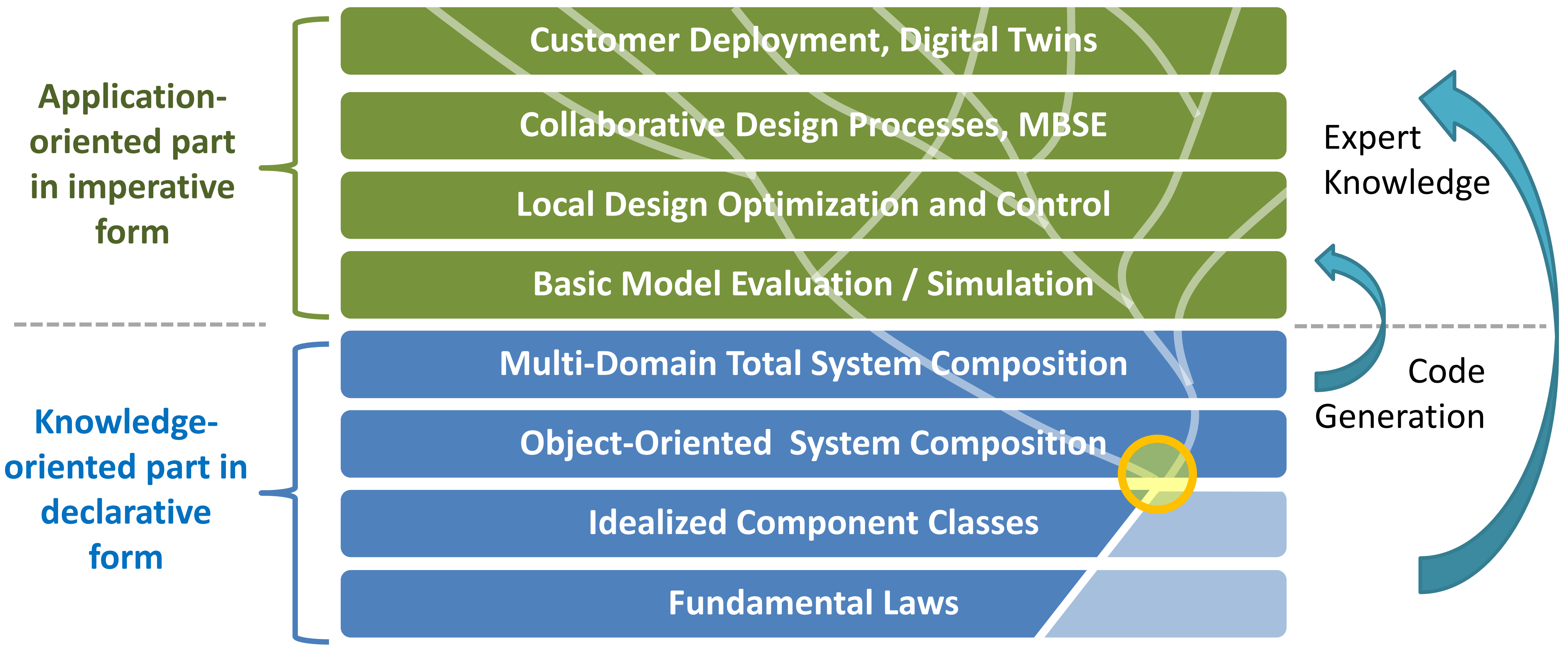

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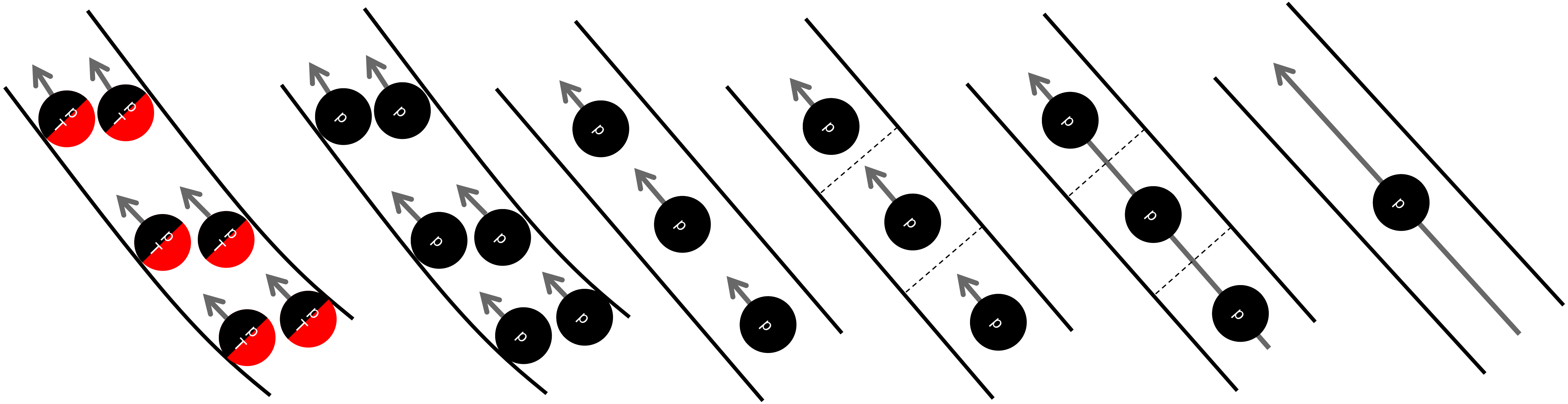
Expert Knowledge
Code Generation

Idealization is the Centerpiece of Modeling



Reduction of Domains → Reduction of Spatial Complexity → Isolation of Interaction → Assuming Instantaneous Equilibrium → Taking One as a Whole

Final result → Hagen Poiseuille $\Delta p = \frac{8\mu L Q}{\pi R^4}$

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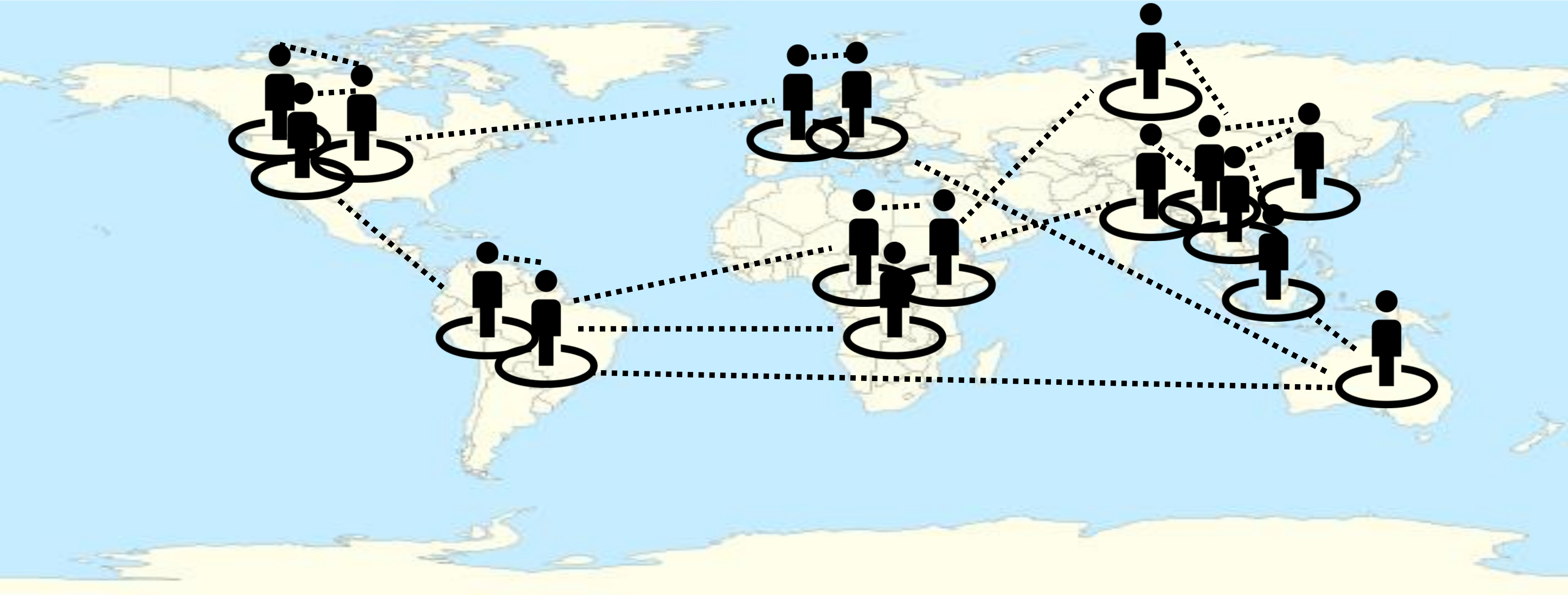

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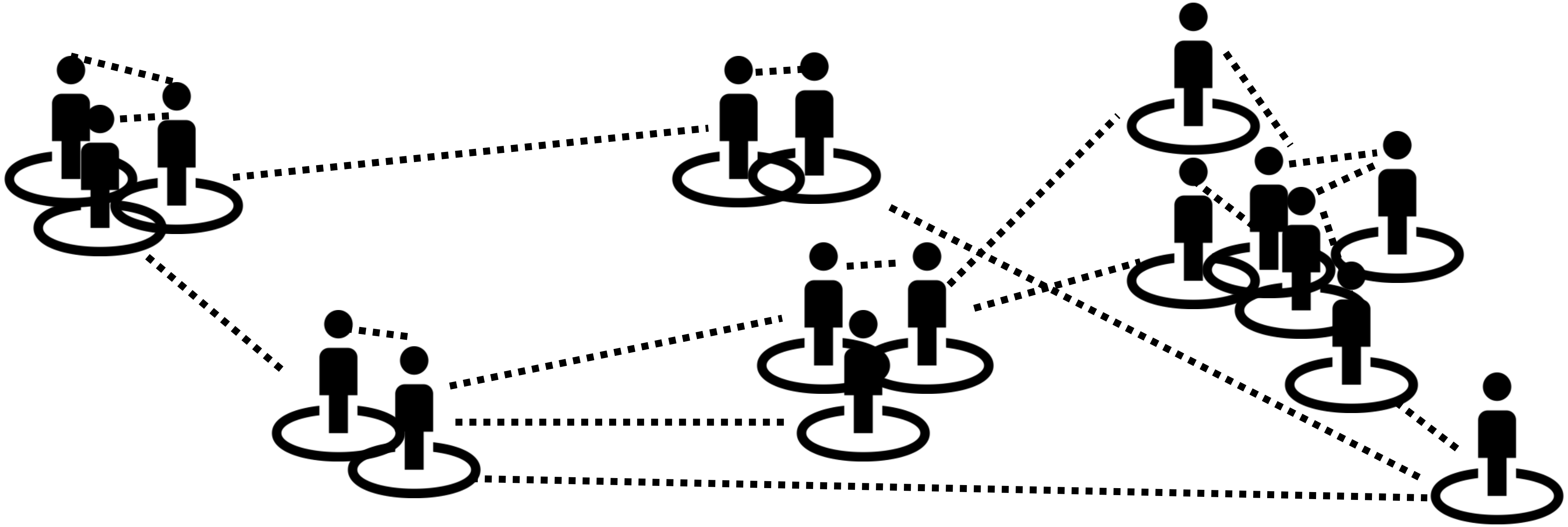

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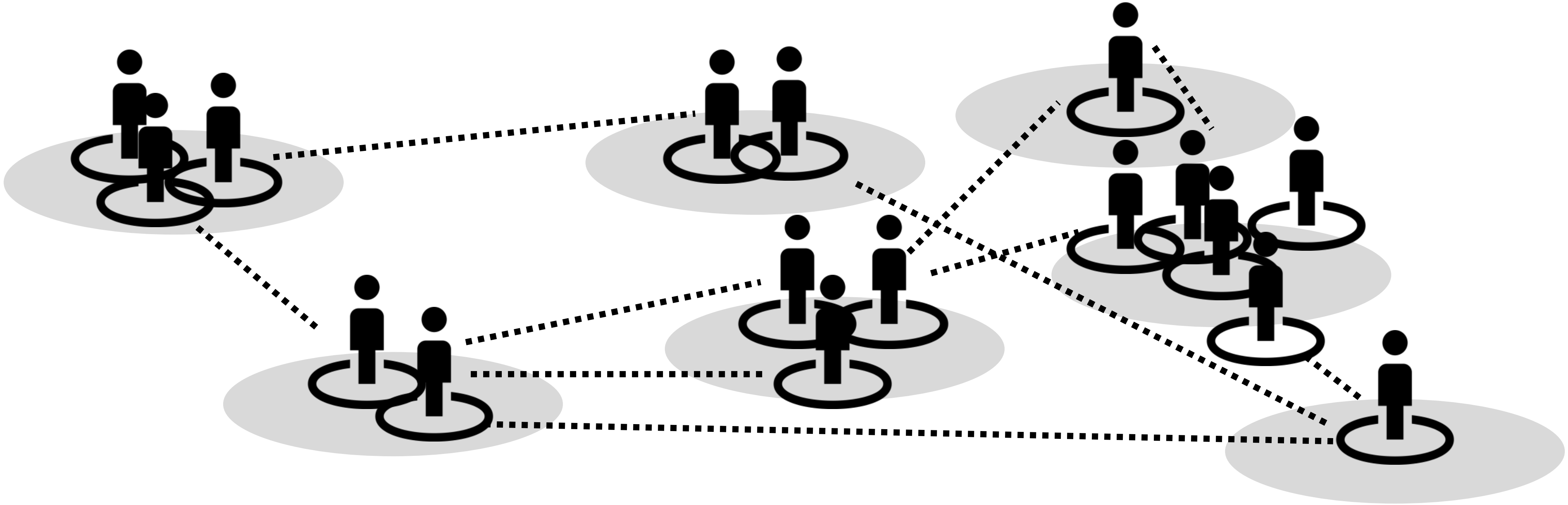

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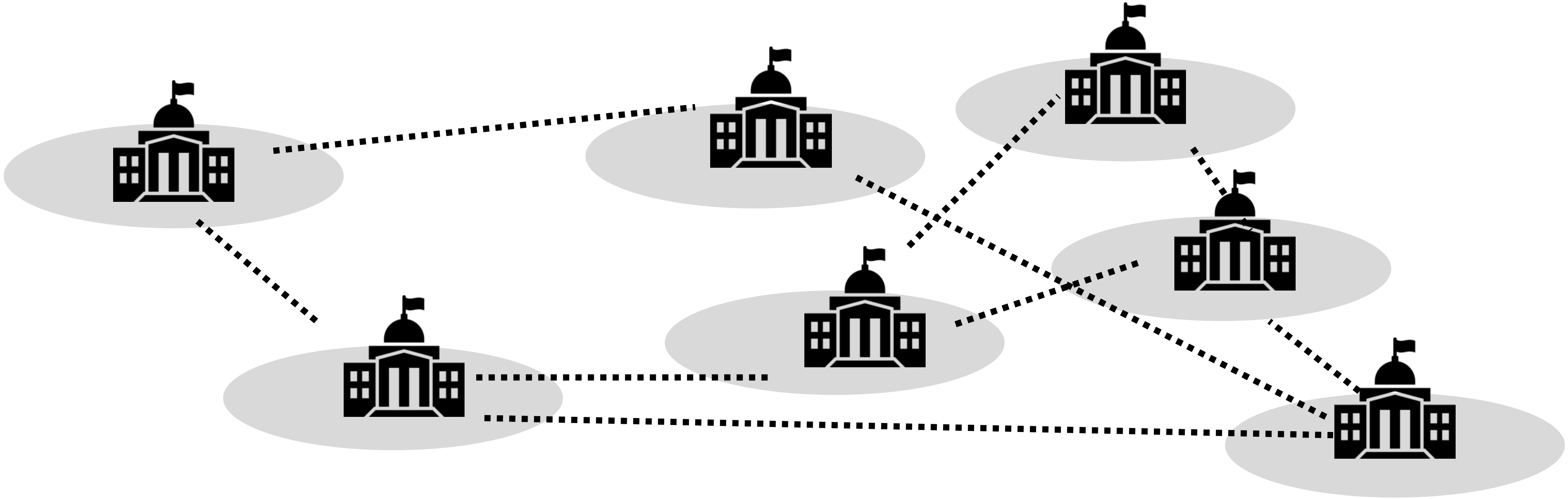

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Idealization provides means to compress reality

Reduction of Domains → Reduction of Spatial Complexity → Isolation of Interaction → Assuming Instanteneous Equilibrium → Taking One as a Whole

Real system → Information Loss (implicit or explicit) → Idealization → Mathematical Model → Simulation

Or taking video compression as analogy

Raw Video Data → Loss function → Compression → Video file → Video encoding

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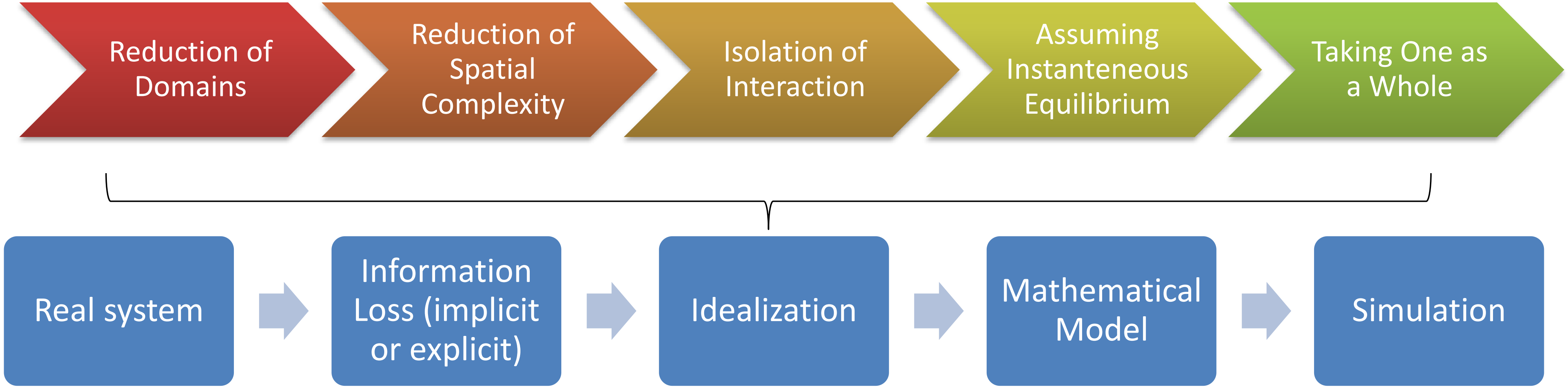
Idealization provides means to compress reality



These fundamental concepts and postulates, which cannot be further reduced logically, form the essential part of a theory, which reason cannot touch. It is the grand object of all theory to make these irreducible elements as simple and as few in numbers as possible.

- John von Neumann, "The Mathematician", 1947

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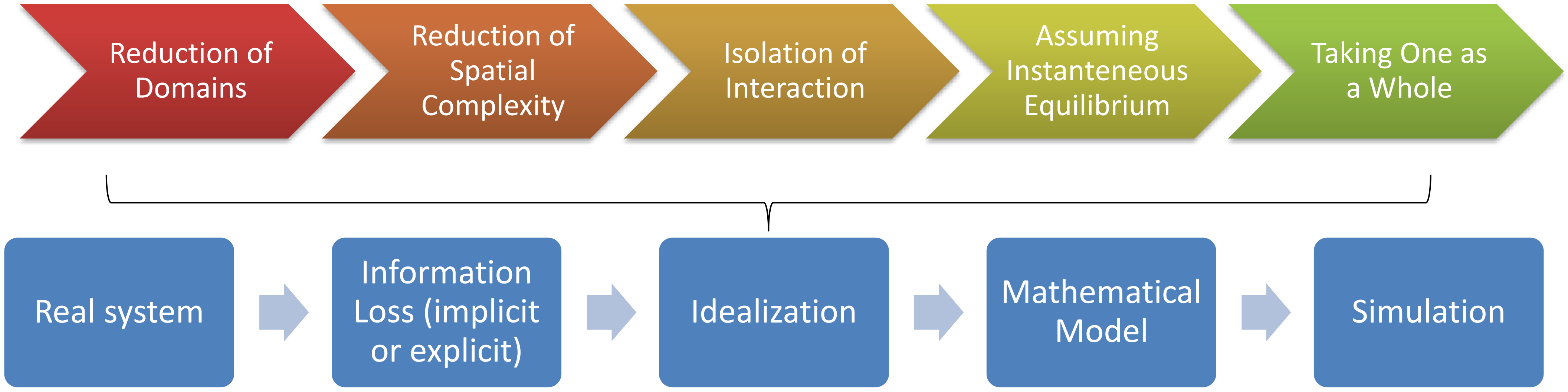


[…] But simplicity certainly reflects what we mean by understanding: ***understanding is compression****. So perhaps this is more about the human mind than it is about the universe.*

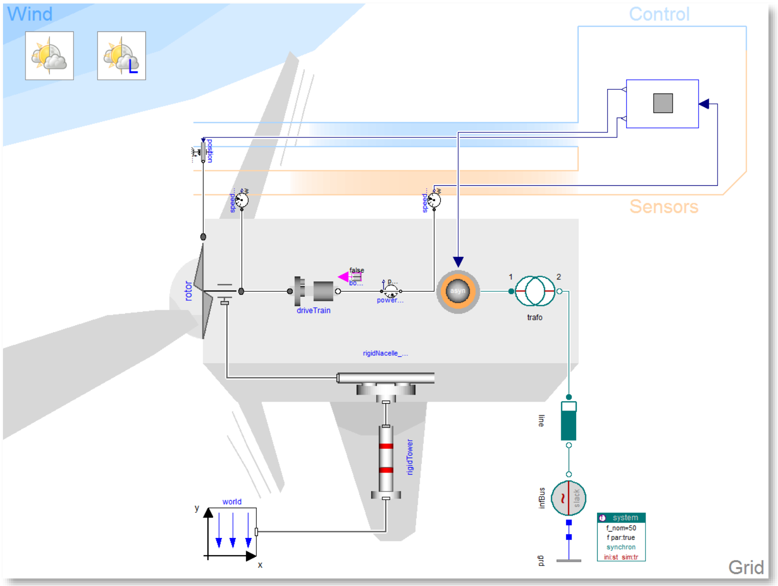
-

- *Gregory J Chaitin,*

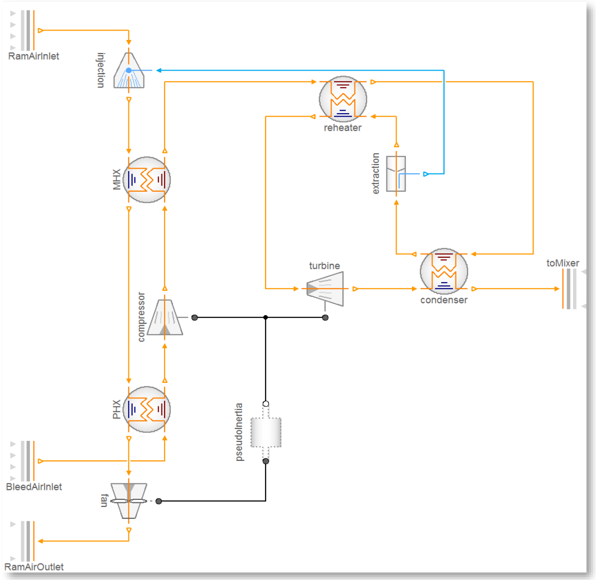
On the intelligibility of the universe and the notions of simplicity, complexity and irreducibility, 2002

Founder of algorithmic information theory

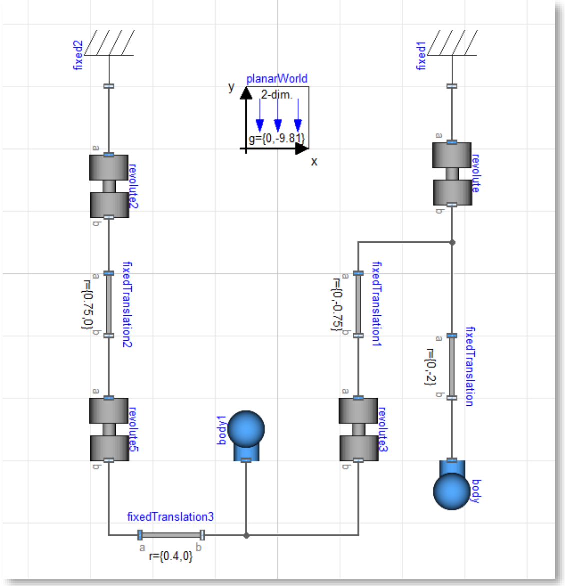
The problem of over-idealization



Fails to achieve instantaneous equivalent of blade element torque and air-flow momentum change

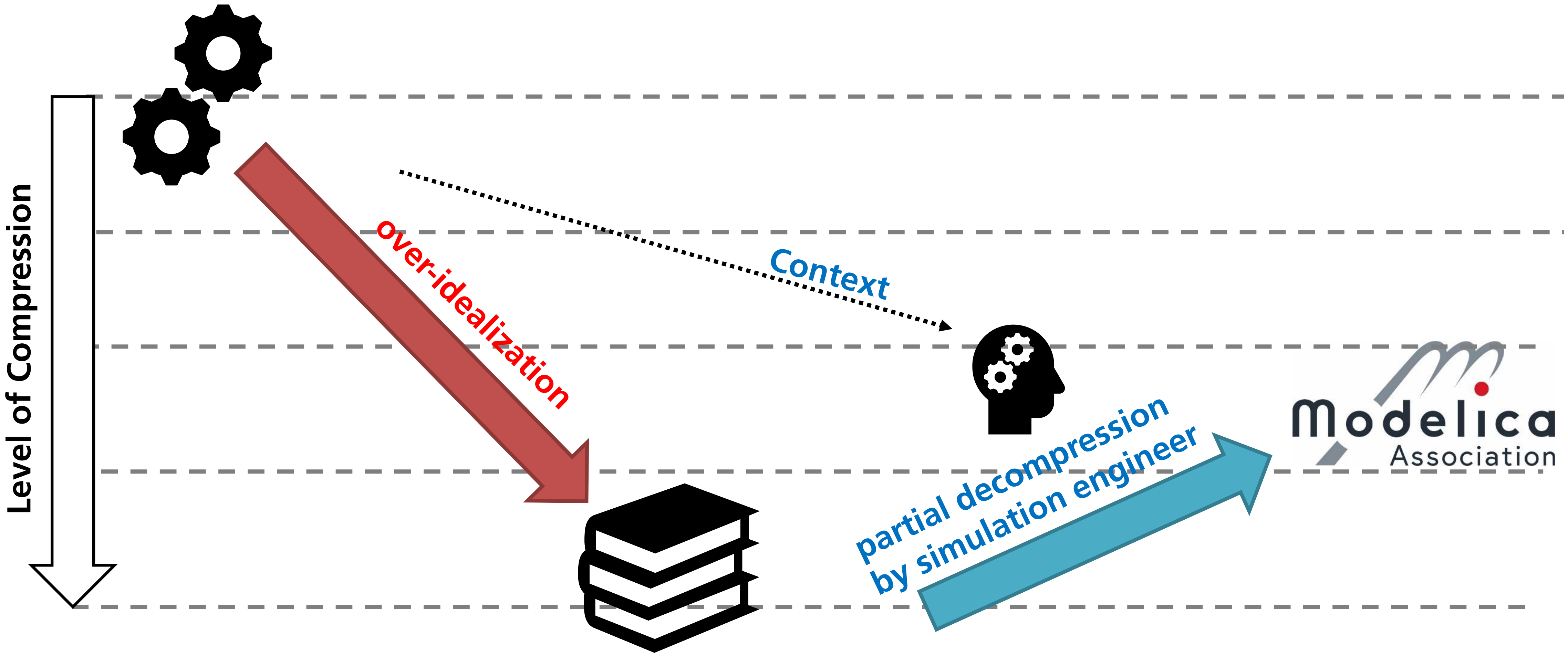


Fails to achieve instantaneous balance of power between compressor, fan and turbine

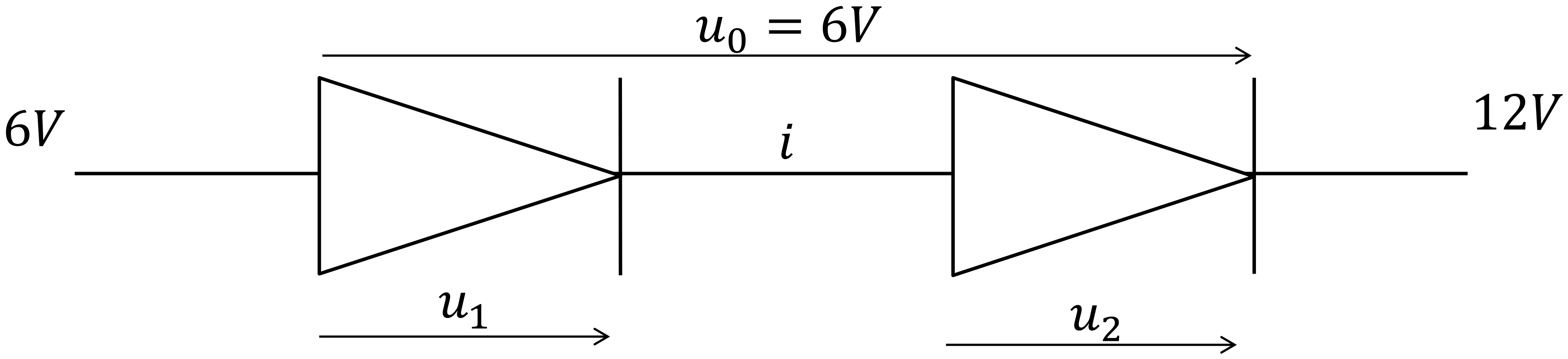


Fails to attribute kinetic energy at point of maximum stretch

Over-idealization is predominant in text books



The problem of over-idealization: idealization in components



$$\underbrace{u_1 = R_1 i \Rightarrow i = G_1 u_1; \qquad u_0 = u_1 + u_2; \qquad u_2 = R_2 i \Rightarrow i = G_2 u_2;}$$

$$u_1 = \frac{G_2 u_0}{G_1 + G_2} = \frac{0 u_0}{0 + 0} \quad \text{Singular!}$$

If we idealize on the component level:

$$\lim_{\Theta \to 0} G_1(\Theta) = \lim_{\Theta \to 0} G_1' \Theta \Rightarrow G_1 = 0 \qquad \lim_{\Theta \to 0} G_2(\Theta) = \lim_{\Theta \to 0} G_2' \Theta \Rightarrow G_2 = 0$$

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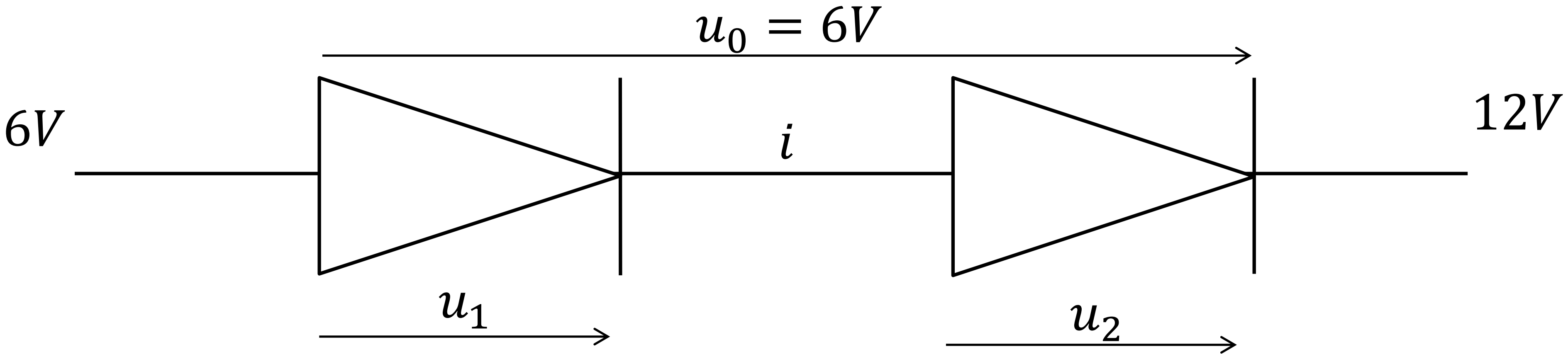

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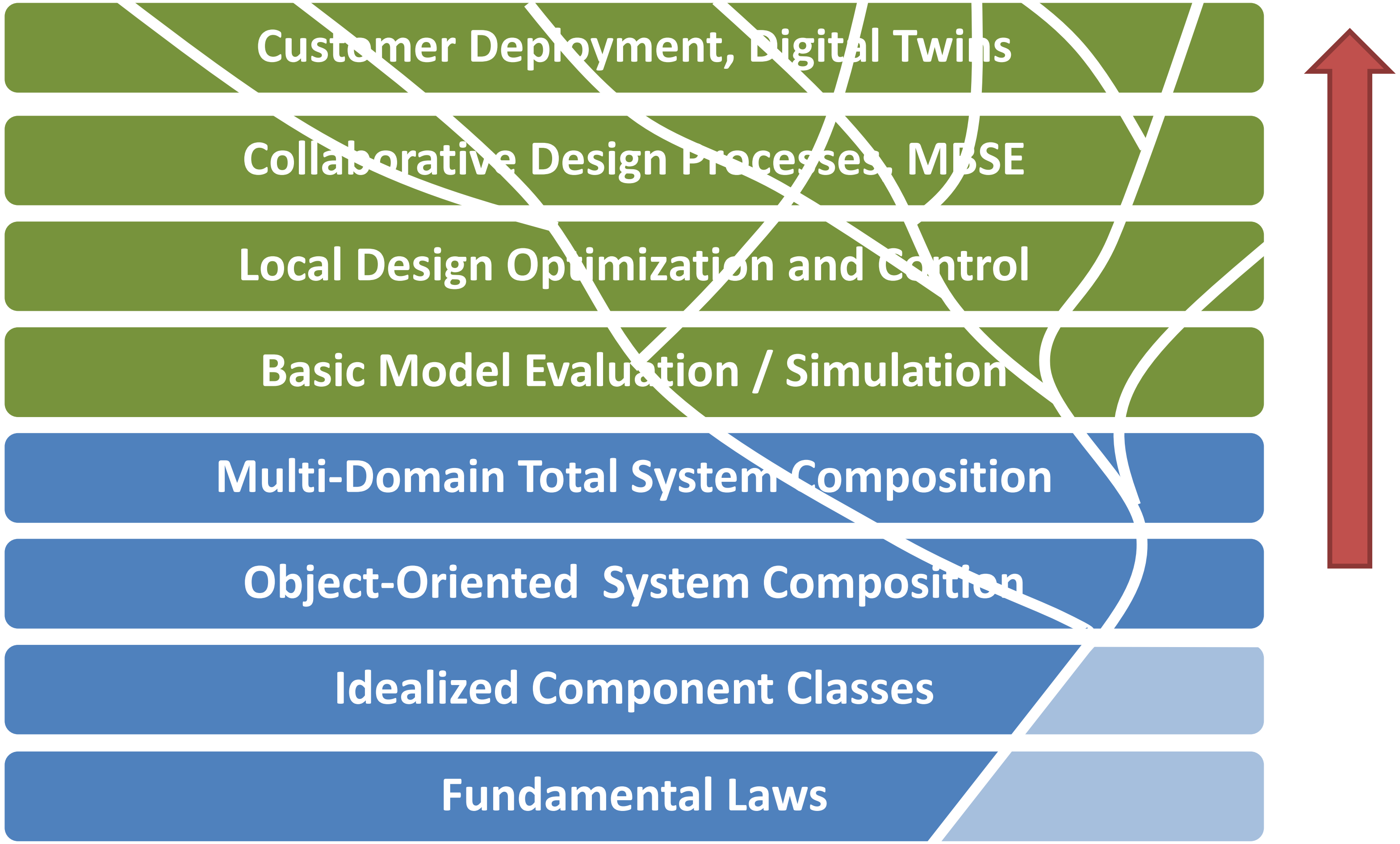
$$u_1 = \frac{G_2 u_0}{G_1 + G_2} = \lim_{\Theta \to 0} \frac{G_2' \Theta u_0}{G_1' \Theta + G_2' \Theta} = \frac{G_2'}{G_1' + G_2'} u_0 \qquad \text{Regular!}$$

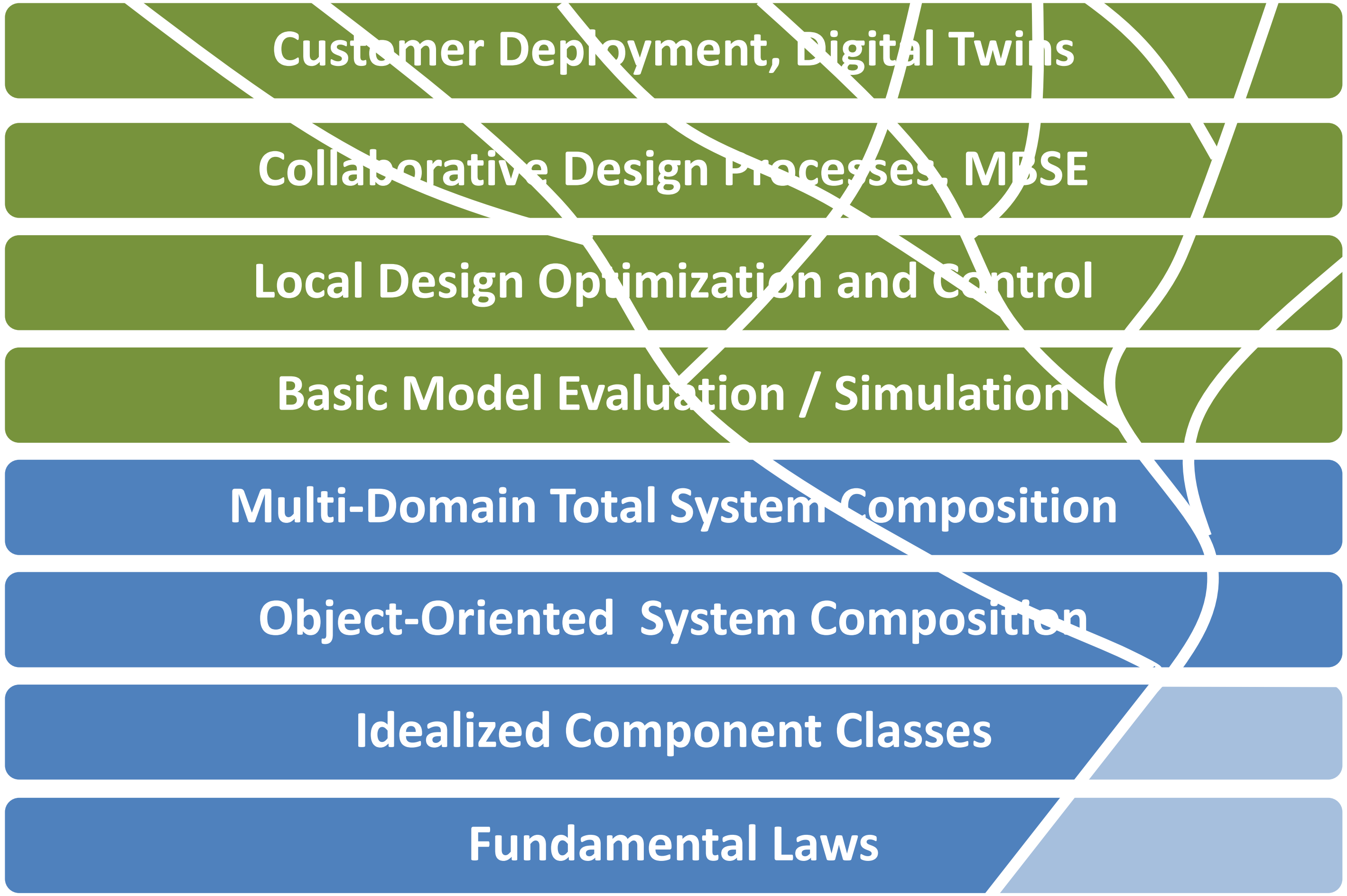
… when idealization is applied on system level.

Over-idealization is the root of our problems.



No tool can recover information that has been lost due to idealization

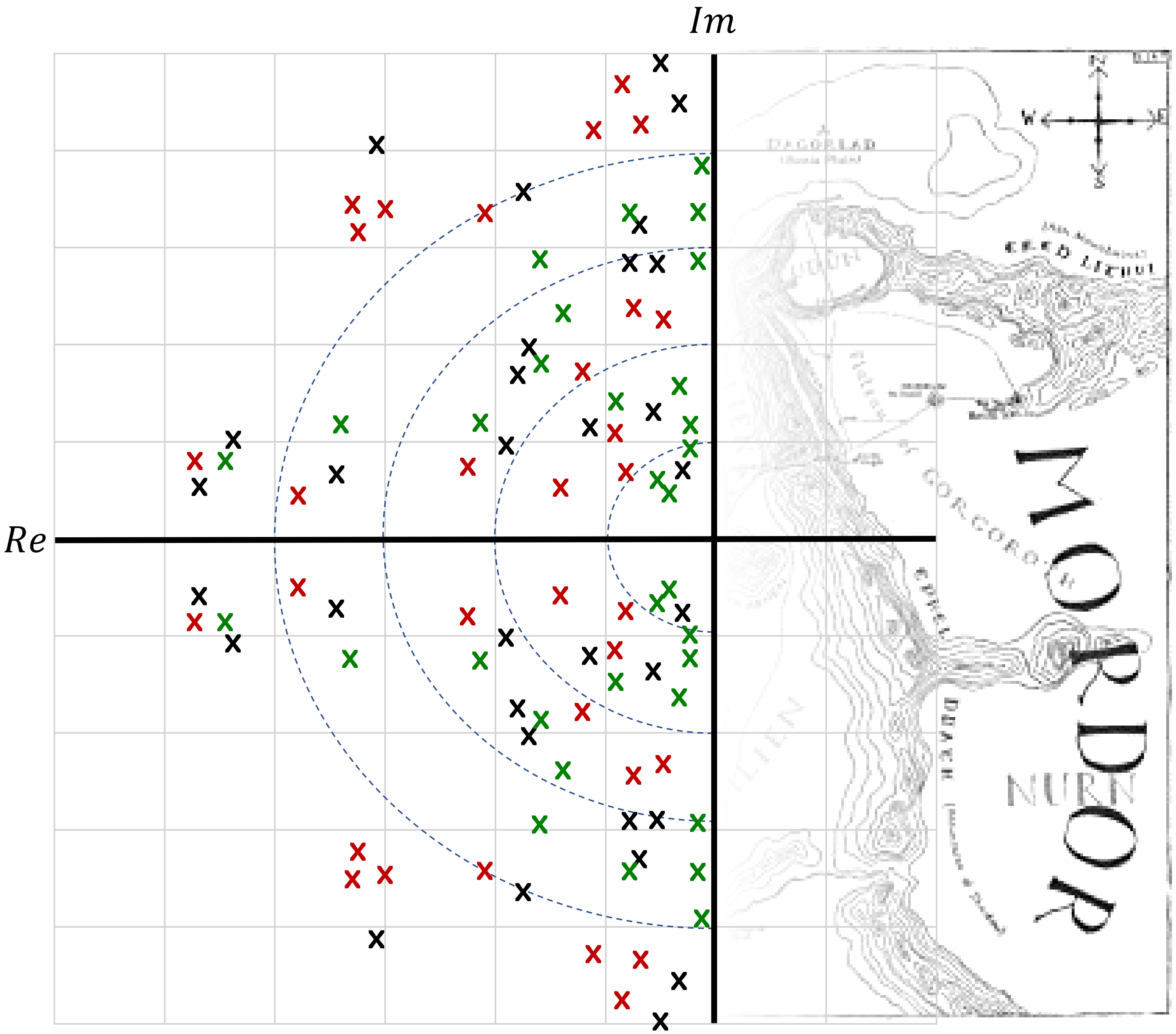
Over-idealization is the root of our problems.



So how to fix the issue?

What do we want to compress?

For system simulation, we aim to compress the **Eigendynamics**



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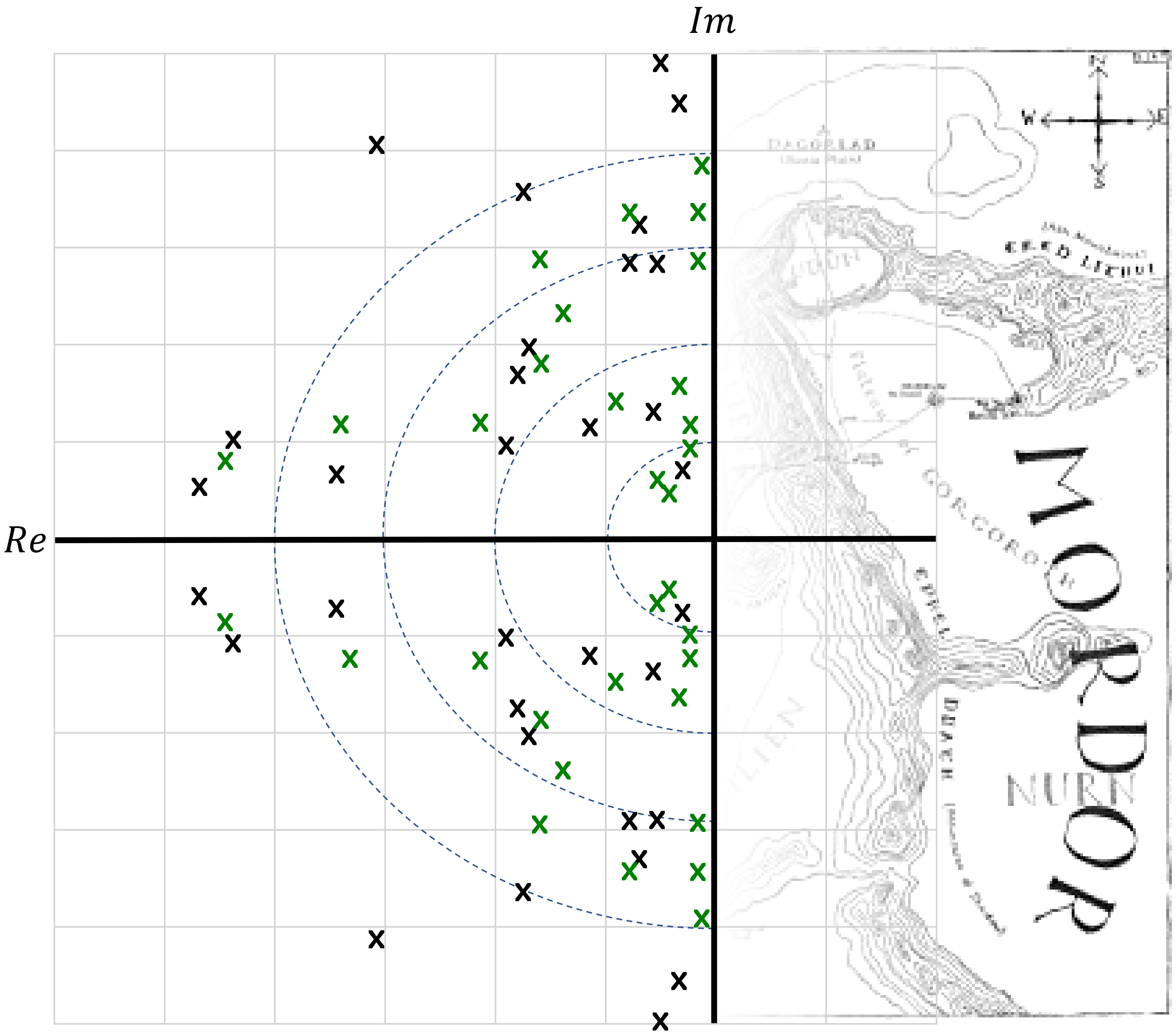
Reduction of Domains



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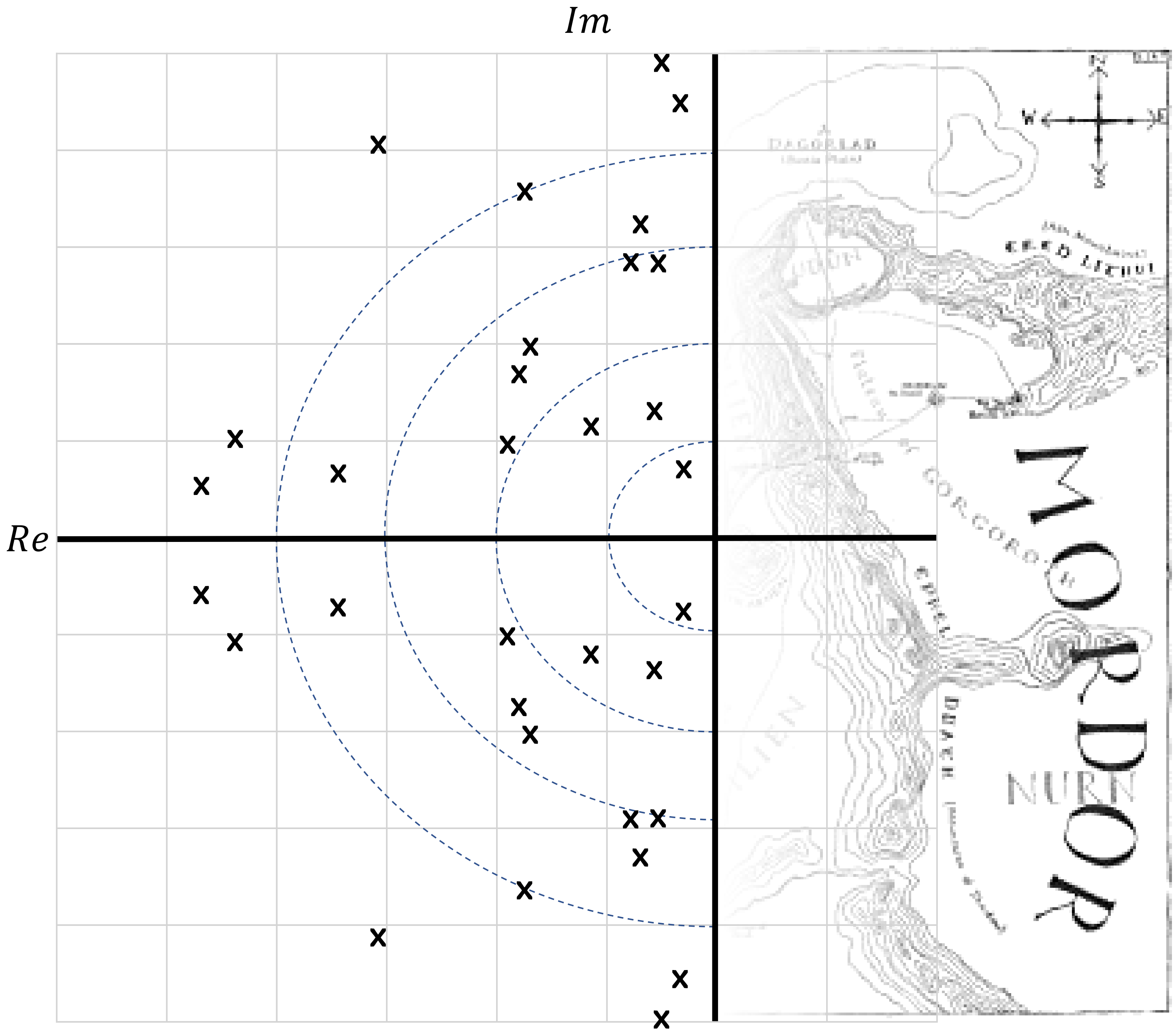
Reduction of Domains

Reduction of Spatial Complexity



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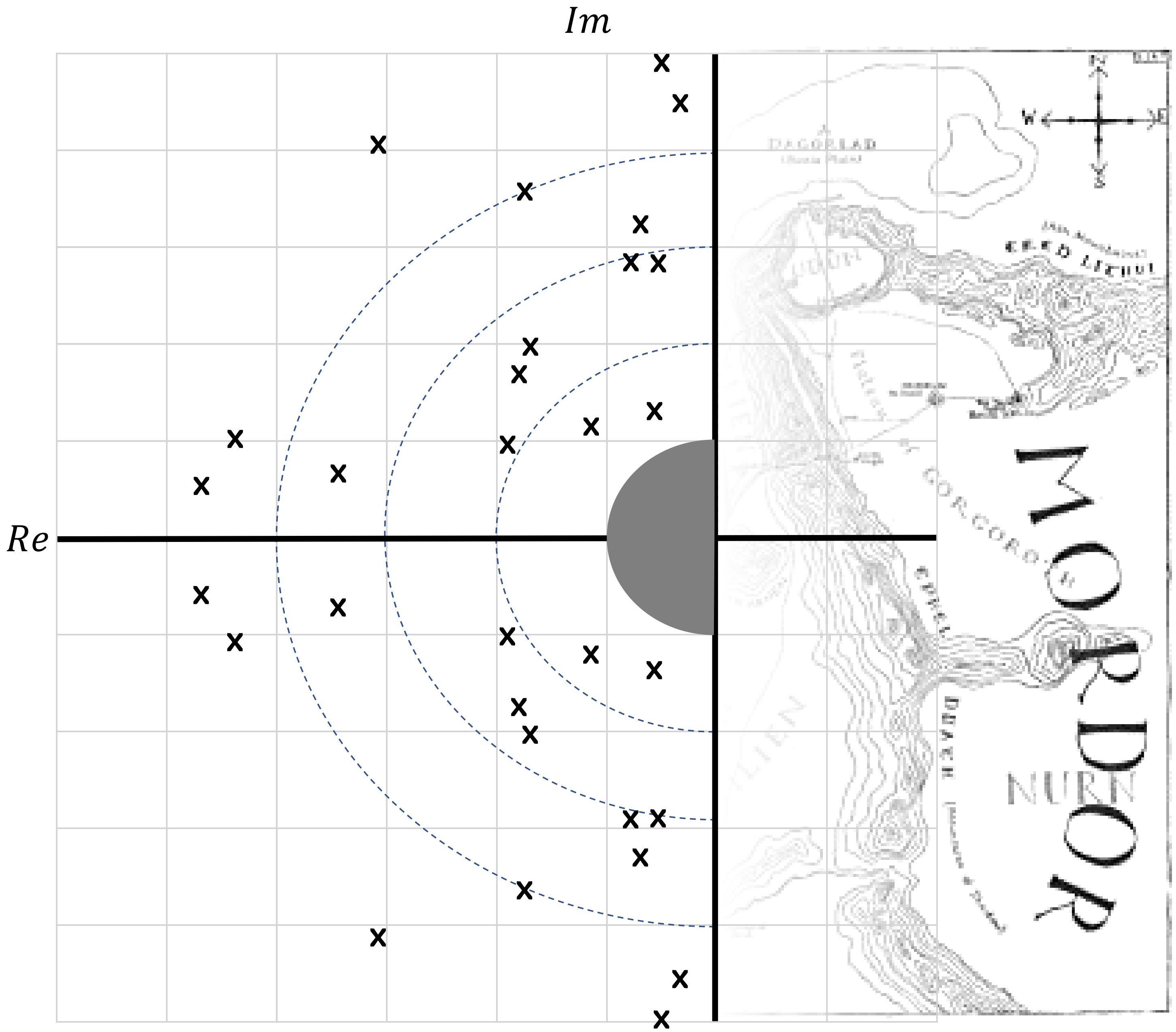
- Isolation of Interaction



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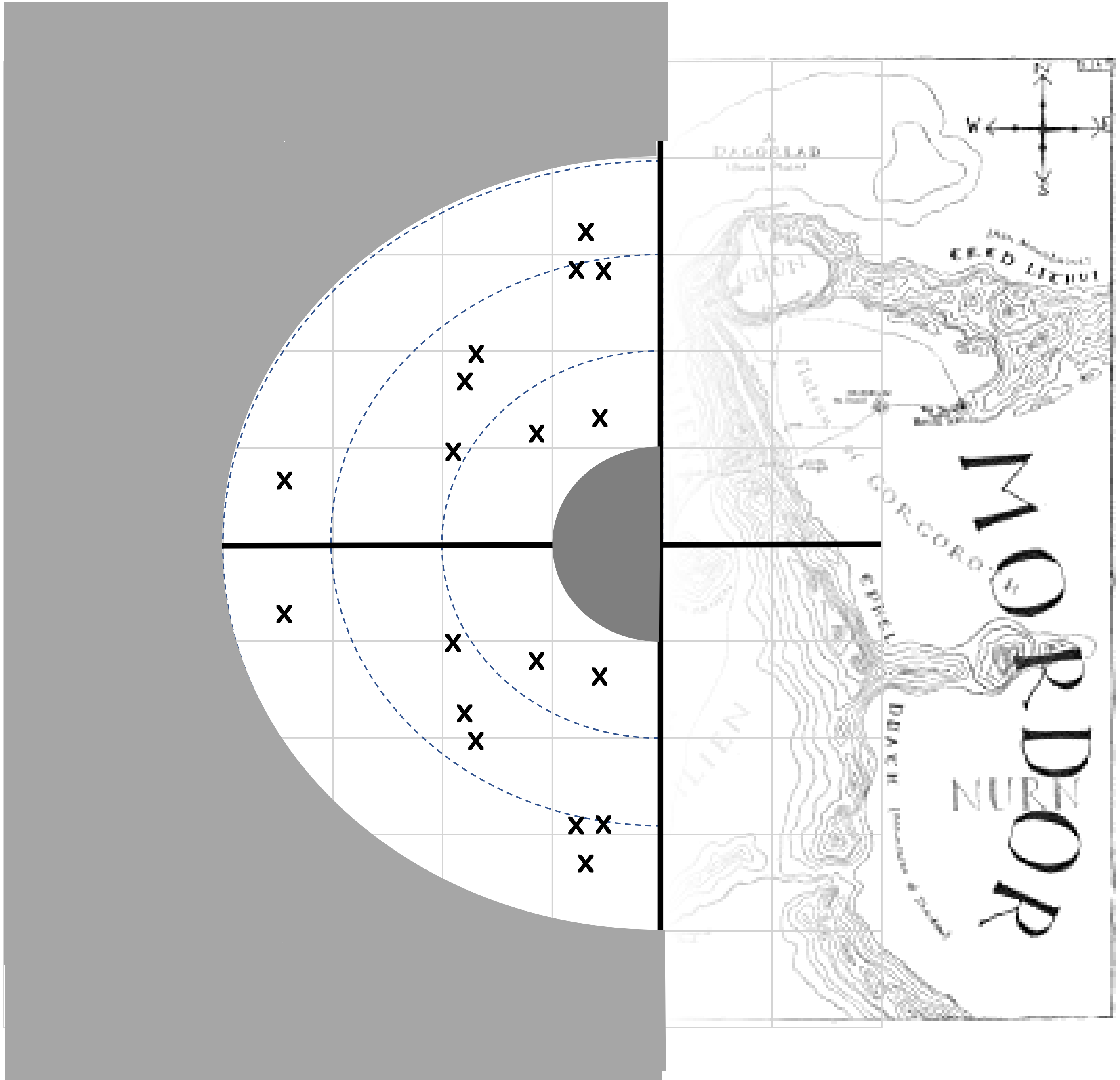

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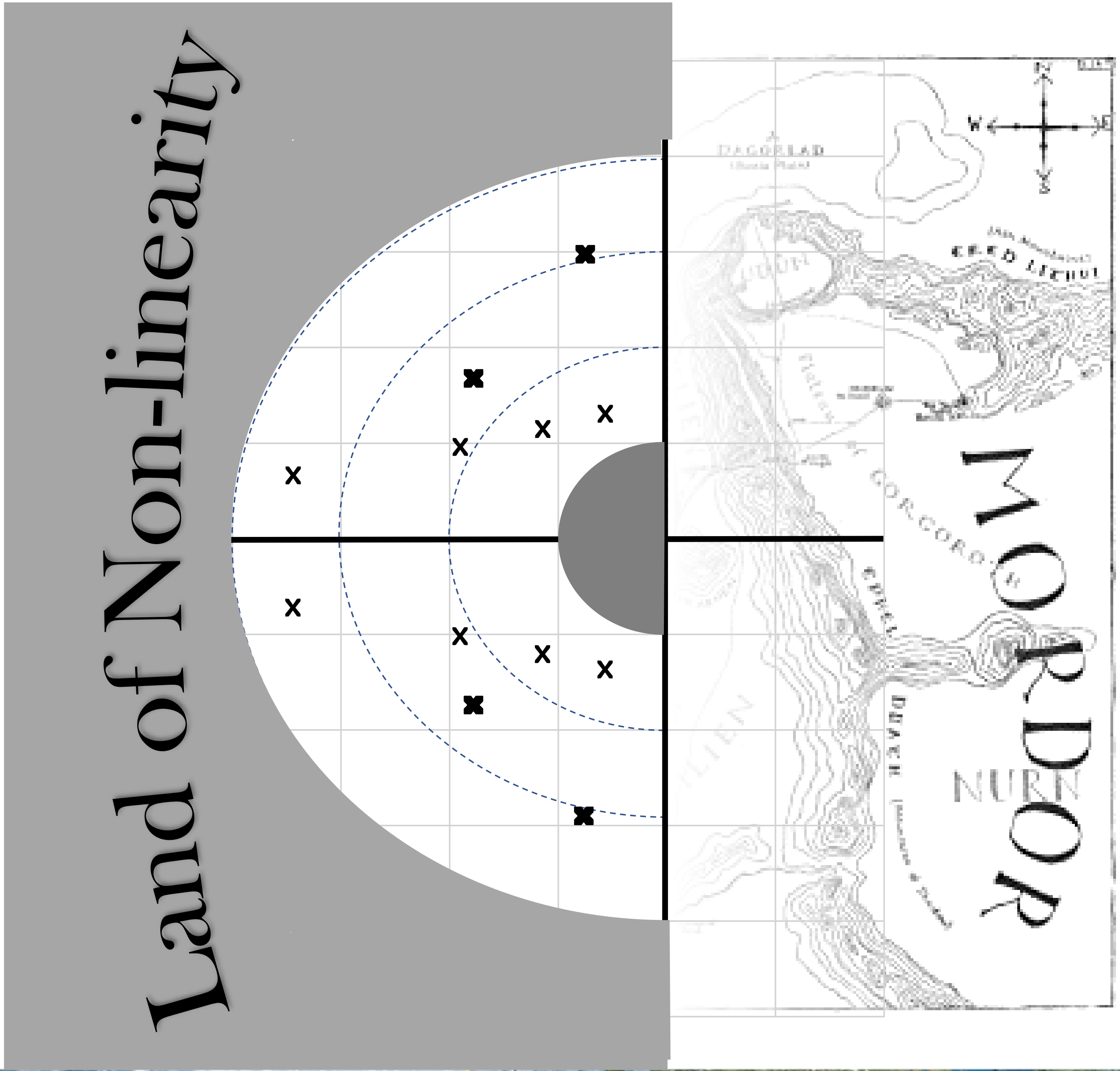
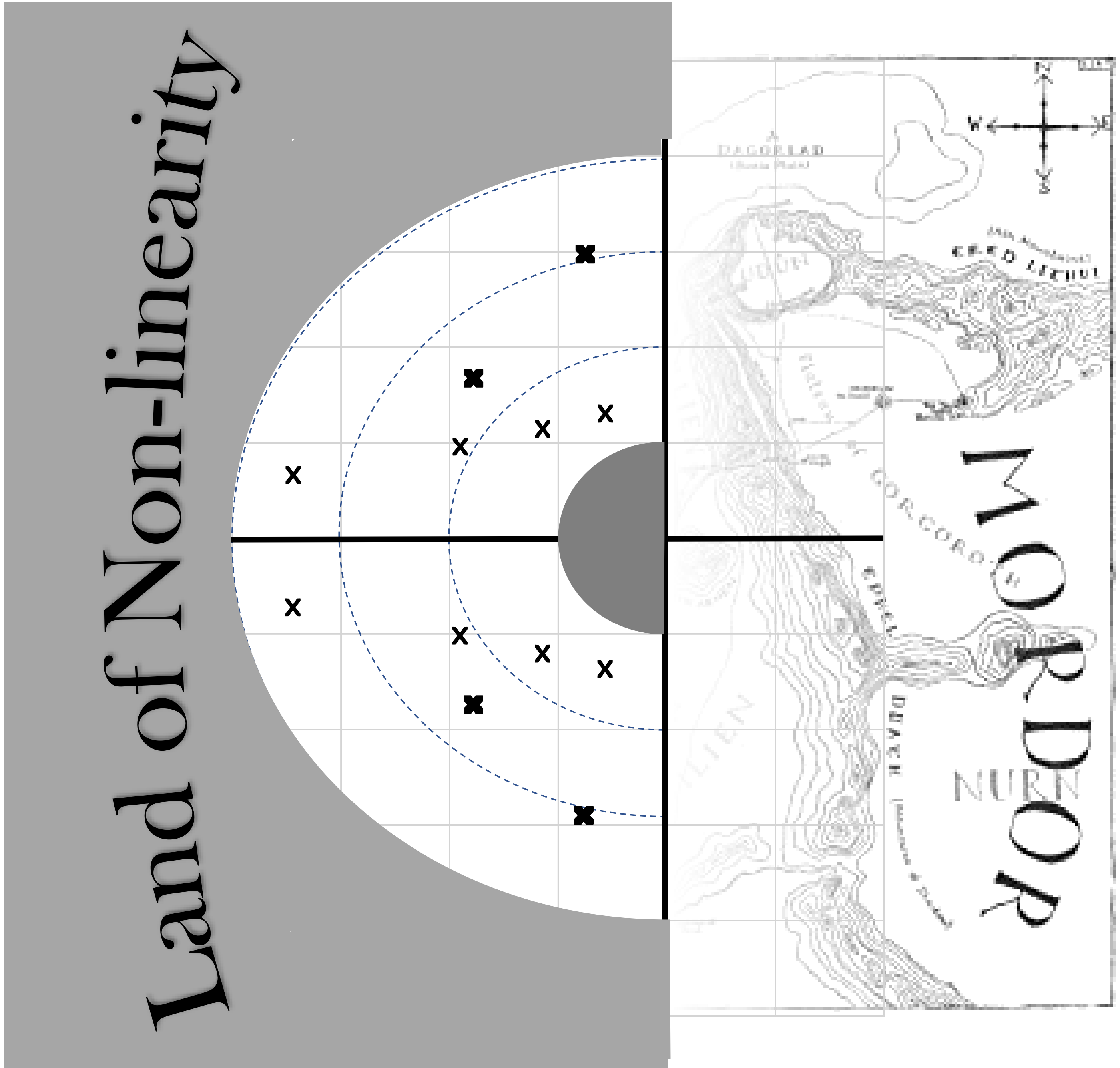


What do we want to compress?

- Hence we end up with differential equations for the dynamic part and algebraic parts to enable compression

- Unfortunately, there is no practical way to ensure the solvability of the non-linear landscape

- Hence, I propose to aim for a different target…



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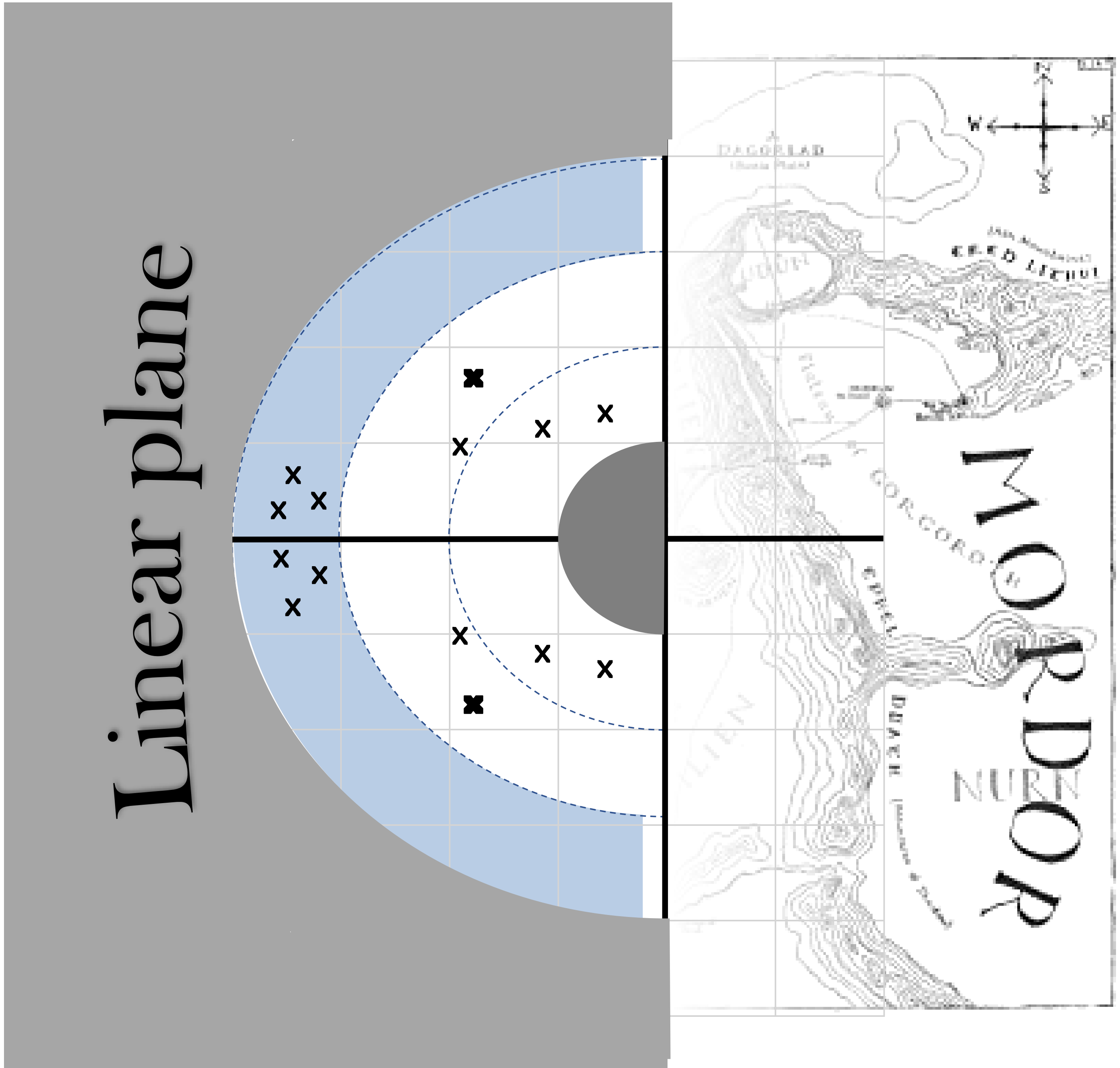
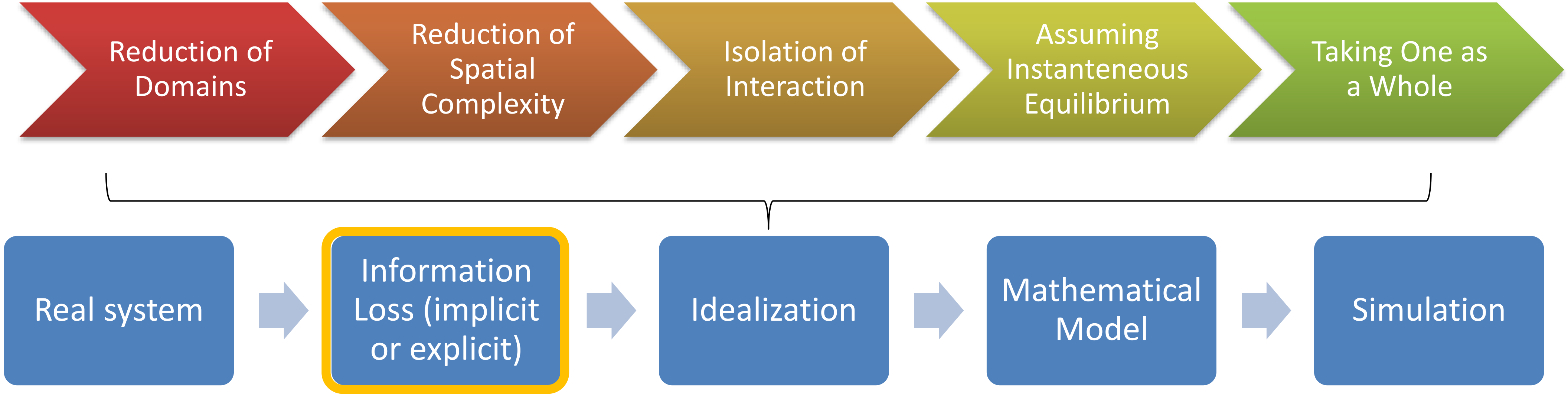
What do we want to compress?

- The implicit system that balances the dynamics shall be only linear, so that it has exactly one solution
- We use auxiliary dynamics (blue) to express the non-linearities that cannot be expressed by the linear part.
- We can find such forms for complex physical system.
- This form is an attractive balance between effectiveness and scalability.



Scalable Idealization of ThermoFluid Streams



An effective compression is mostly about making a deliberate error.

We shall look for an error that helps to compress a lot but impacts the simulation result only by little

How to do that for thermofluid streams?

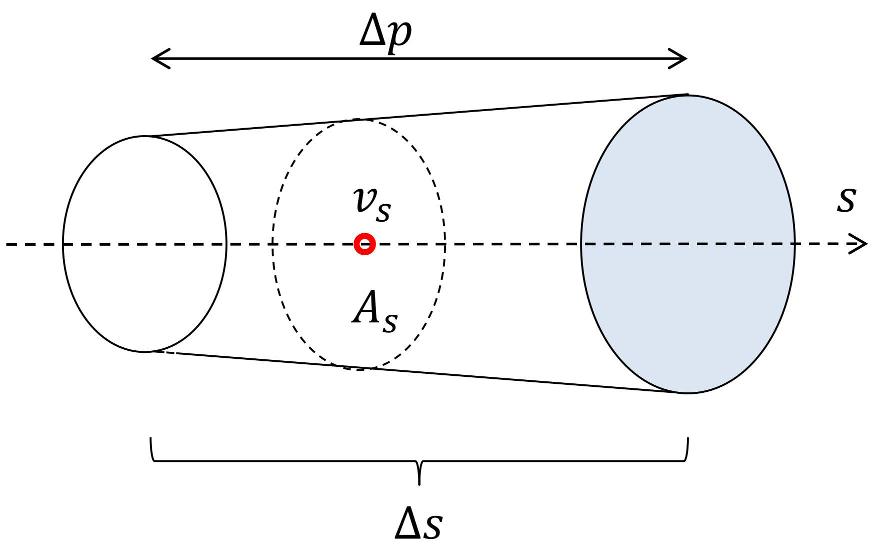
Starting from Euler’s law for 1D fluid stream

- Let us start with Euler‘s equation

$$\rho \frac{\partial v_s}{\partial t} + \rho \, v_s \frac{\partial v_s}{\partial s} = -\frac{\partial p}{\partial s} - \frac{\partial p_{ext}}{\partial s}$$

- And bring it into integral form for an arbitrary pipe section

$$\int \rho \frac{\partial v_s}{\partial t} ds + \rho \, \bar{v} \, \Delta v = -\Delta p - \Delta p_{ext}$$



Starting from Euler's law for 1D fluid stream

- We can substitute v_s by the mass flow rate:

$$v_s = \frac{\dot{m}}{\rho A_S}$$

- Assuming the mass flow rate is constant over the stream line, we can take it out of the integral.

- The **inertance** L is defined as

$$L = \int \frac{1}{A_S} ds$$

- And the **inertial pressure** r is defined by

$$\Delta r = L \frac{d\dot{m}}{dt}$$

$$\int \rho \frac{\partial v_s}{\partial t} ds + \rho \bar{v} \, \Delta v = -\Delta p - \Delta p_{ext}$$

$$\frac{d\dot{m}}{dt} \int \frac{1}{A_s} ds + \Delta q = -\Delta p - \Delta p_{ext}$$

$$\Delta r + \Delta q = -\Delta p - \Delta p_{ext}$$

Starting from Euler’s law for 1D fluid stream

- Let us define the **steady mass flow pressure** $\hat{p}$

$$p = \hat{p} + r$$

- It is defined as the complement to the inertial pressure. Plugging in the definition…
- … and rearranging…
- … let us remind that q and p_{ext} depend on the thermodynamic state of the medium

$$\Delta r + \Delta q = -\Delta p - \Delta p_{ext}$$

$$\Delta q = -\Delta \hat{p} - \Delta p_{ext}$$

$$\Delta \hat{p} = \Delta p_{ext} + \Delta q$$

$$\Delta \hat{p} = \Delta p_{ext}(p, \dot{m}, \dots) + \Delta q(p, \dot{m}, \dots)$$

Starting from Euler’s law for 1D fluid stream

$$\Delta\hat{p} \;=\; \Delta p_{ext}(\hat{p}, \dot{m}, \dots) + \Delta q(\hat{p}, \dot{m}, \dots)$$

On ne voit bien qu'avec le cœur. L'essentiel est invisible pour les yeux

source: Le petit prince

Starting from Euler's law for 1D fluid stream

- Using $\hat{p}$ instead of p is acceptable because:
  - when $d\dot{m}/dt = 0$, the error is zero
  - for gases, r is typically small
  - for liquids, the RHS is typically insensitive to r
  - many formulas (as for friction) anyway assume steady mass flow…
  - One may also say that $\hat{p}$ and r have a different spatial resolution.
- The scheme is applicable to 96.3% of our typical Modelica use cases…

$$\Delta\hat{p} = \Delta p_{ext}(\hat{p}, \dot{m}, \ldots) + \Delta q(\hat{p}, \dot{m}, \ldots)$$

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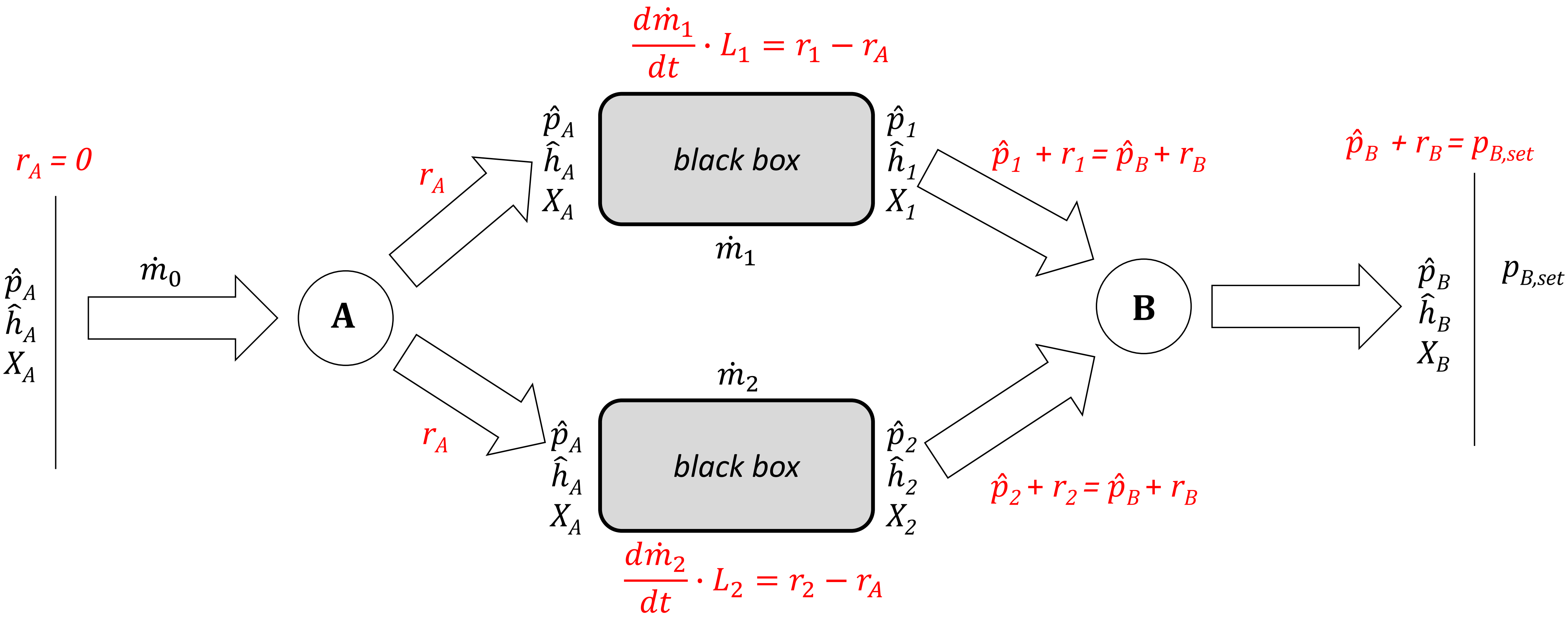
$$\Delta\hat{p} = \Delta p_{ext}(\hat{p}, \dot{m}, \dots) + \Delta q(\hat{p}, \dot{m}, \dots)$$

Real system → Information Loss (implicit or explicit) → Idealization

DLR

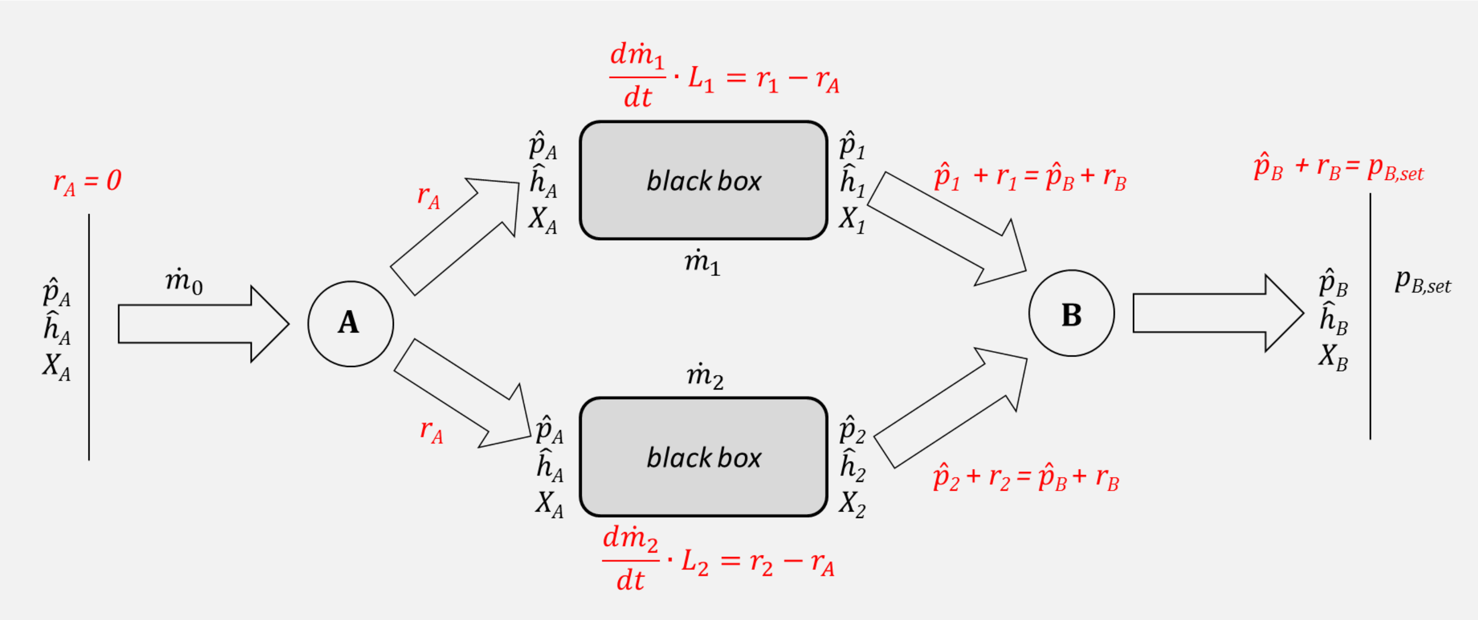
A First System: Resulting Equations



The black equations can computed straight-forward downstream

What about the red equations? How to get: $d\dot{m}_1/dt$, r_1 $d\dot{m}_2/dt, r_2$

A first circuit: resulting equations



We can directly extract:

- $\frac{d\dot{m}_1}{dt} \cdot L_1 = r_1 - r_A = r_1$
- $\frac{d\dot{m}_2}{dt} \cdot L_2 = r_2 - r_A = r_2$

The B junction results in:

- $\hat{p}_1 + r_1 = \hat{p}_2 + r_2$

index-reduction over junction A yields:

- $d\dot{m}_1/dt = -d\dot{m}_2/dt$

from $\dot{m}_1 + \dot{m}_2 = \dot{m}_0$, $\dot{m}_0$ given

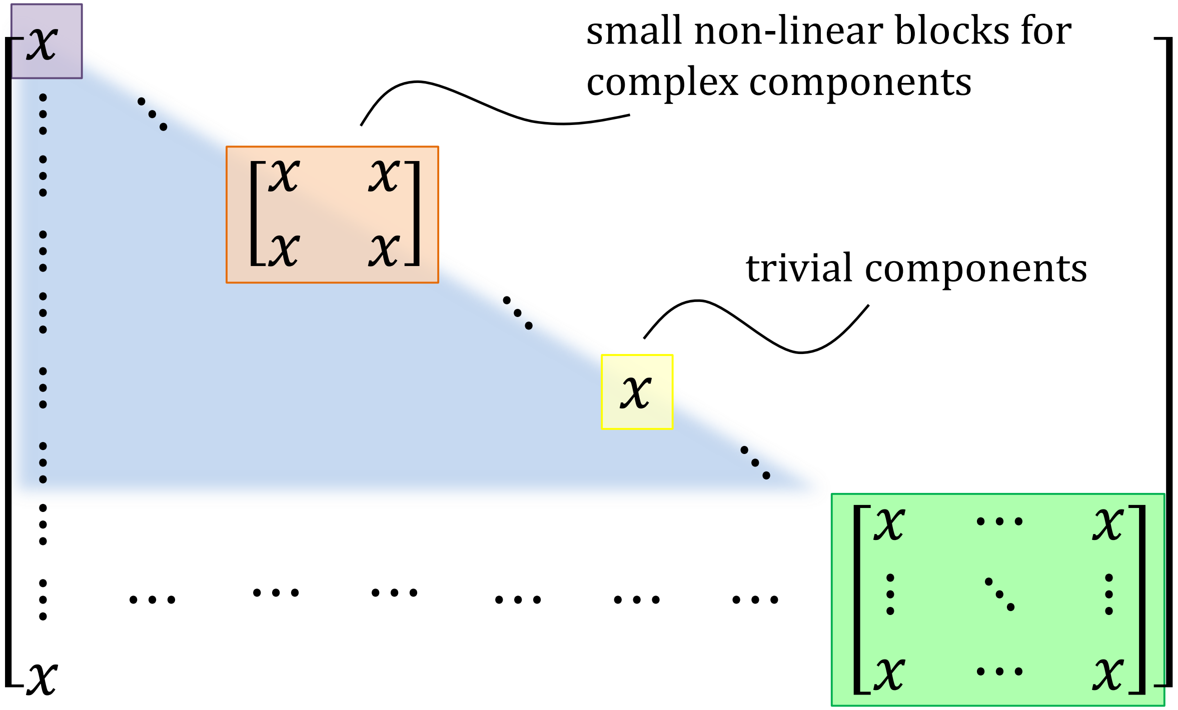
$$\begin{bmatrix} -1 & & L_1 & \\ & -1 & & L_2 \\ 1 & -1 & & \\ & & 1 & -1 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ d\dot{m}_1/dt \\ d\dot{m}_2/dt \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \hat{p}_2 - \hat{p}_1 \\ 0 \end{bmatrix}$$

- Simple linear equation system
- Matrix has only constant entries
- Can be inverted upfront
- Dymola does that!

Downstream computation of thermodynamic state, assuming

Computation of mass-flow change using linear equation system over r (mostly upfront invertible)

Resulting BLT Form

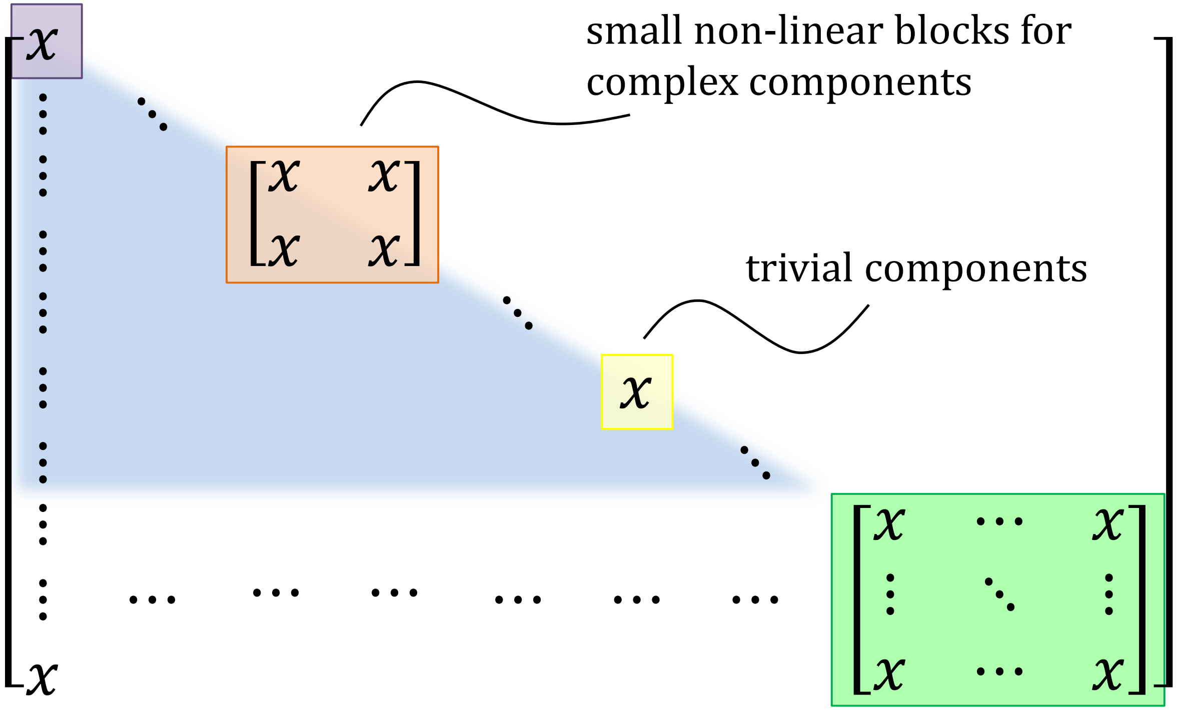


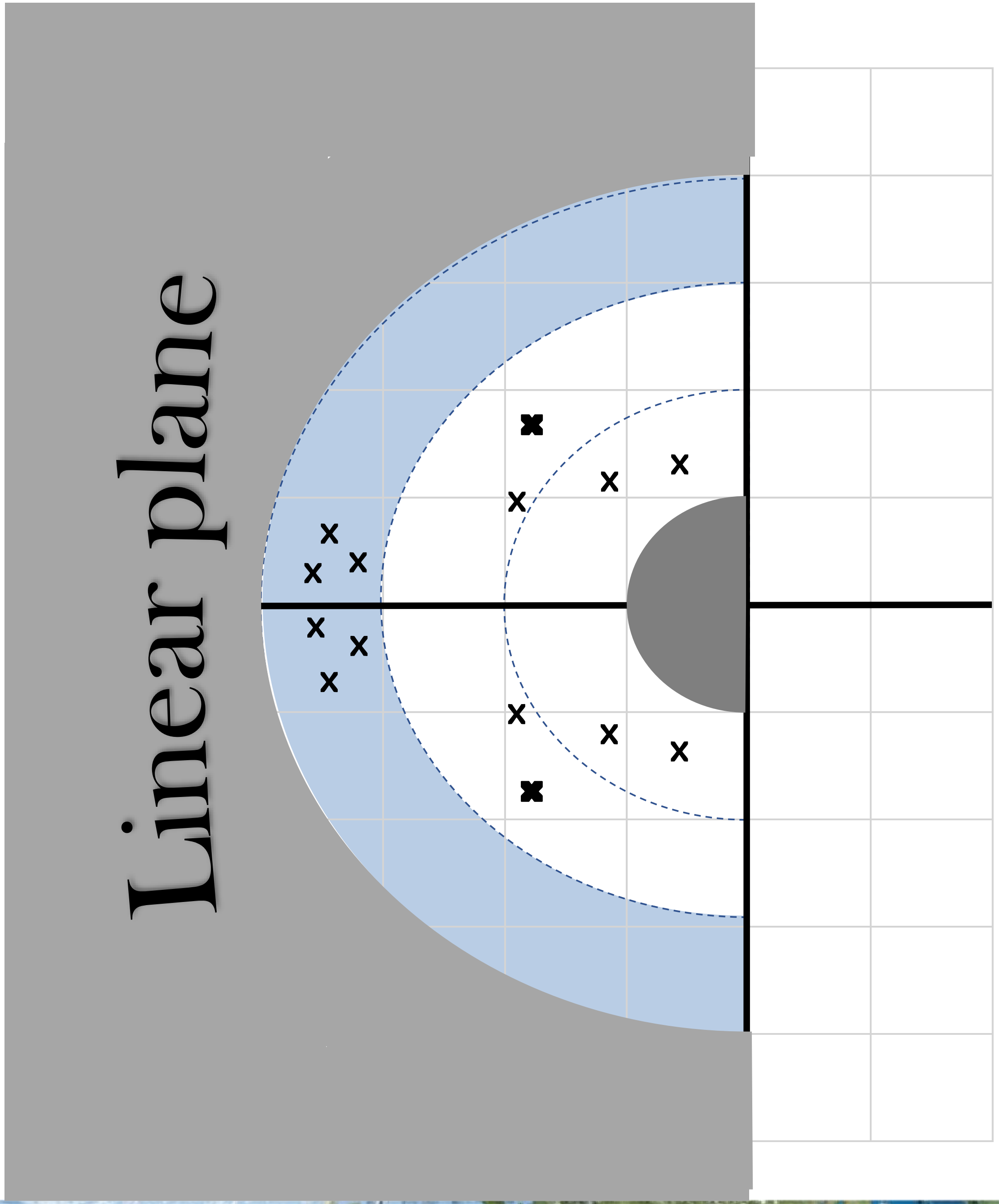
- The large block at the end is linear and invertible upfront

Robus component models
➔
Robust system model

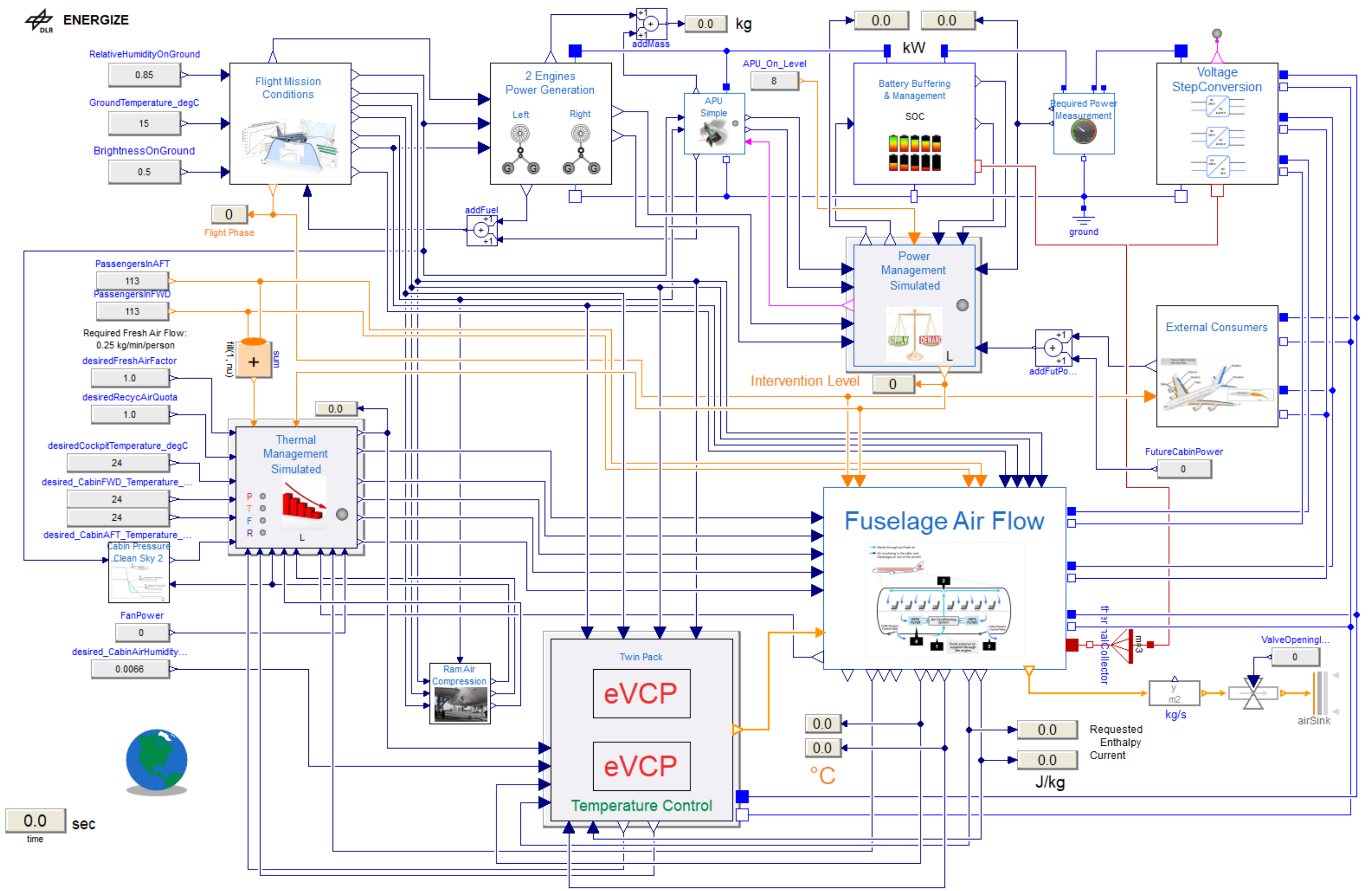
- Size of the linear block proportional to number of branches

Resulting BLT Form





And this method scales…



- The ENERGIZE Model describes a single-aisle more-electric aircraft
- It contains a combines electrical power and thermal management
- It allows the simulation of full flight missions under a wide variety of climatic conditions
- > 18,000 equations
- > 300 states
- faster than real-time

Positioning of the approach

Robustnes

Finite Volume Approach:
- No non-linear equation systems
- Many states
- Stiffness
- High-Frequency Oscillations

ODE

DAE

DLR ThermoFluid Stream:
- No non-linear equation systems
- Few states
- Manipulable Stiffness
- Manipulable Frequency

AE

Algebraic Stream Approach:
- Complicated non-linear equation systems
- No states at all or very few
- Non-stiff / no oscillations

Performance

DLR

How about mechanics?

Robustnes

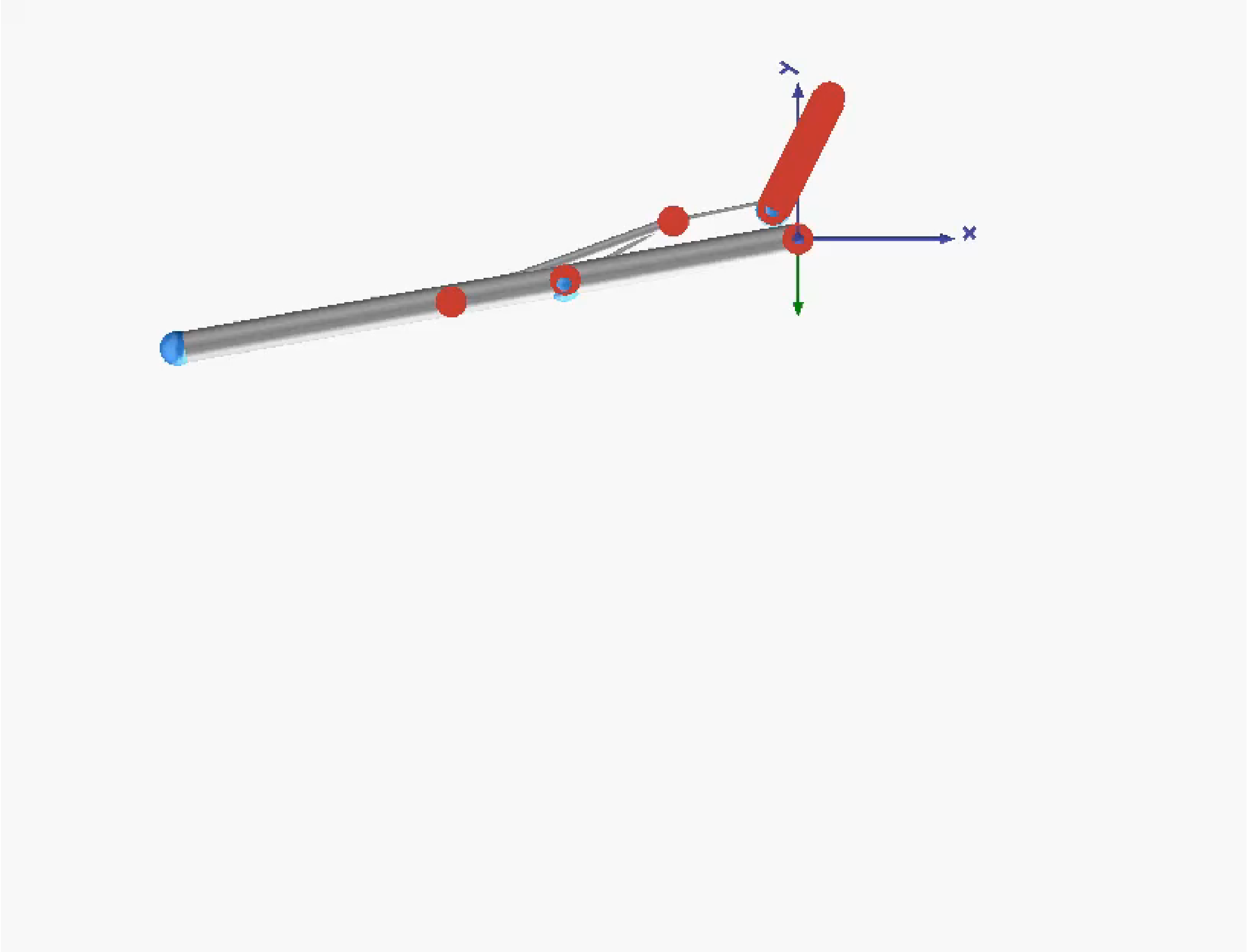
Mass-Spring Approach
- No non-linear equation systems
- Many states
- Stiffness
- High-Frequency Oscillations

?

Rigid Multibody
- Few states
- Very efficient
- Non-linear equation systems
- Not feasible for contacts

Performance

Reformulation of multibody dynamics



Again all non-linear computations are explict

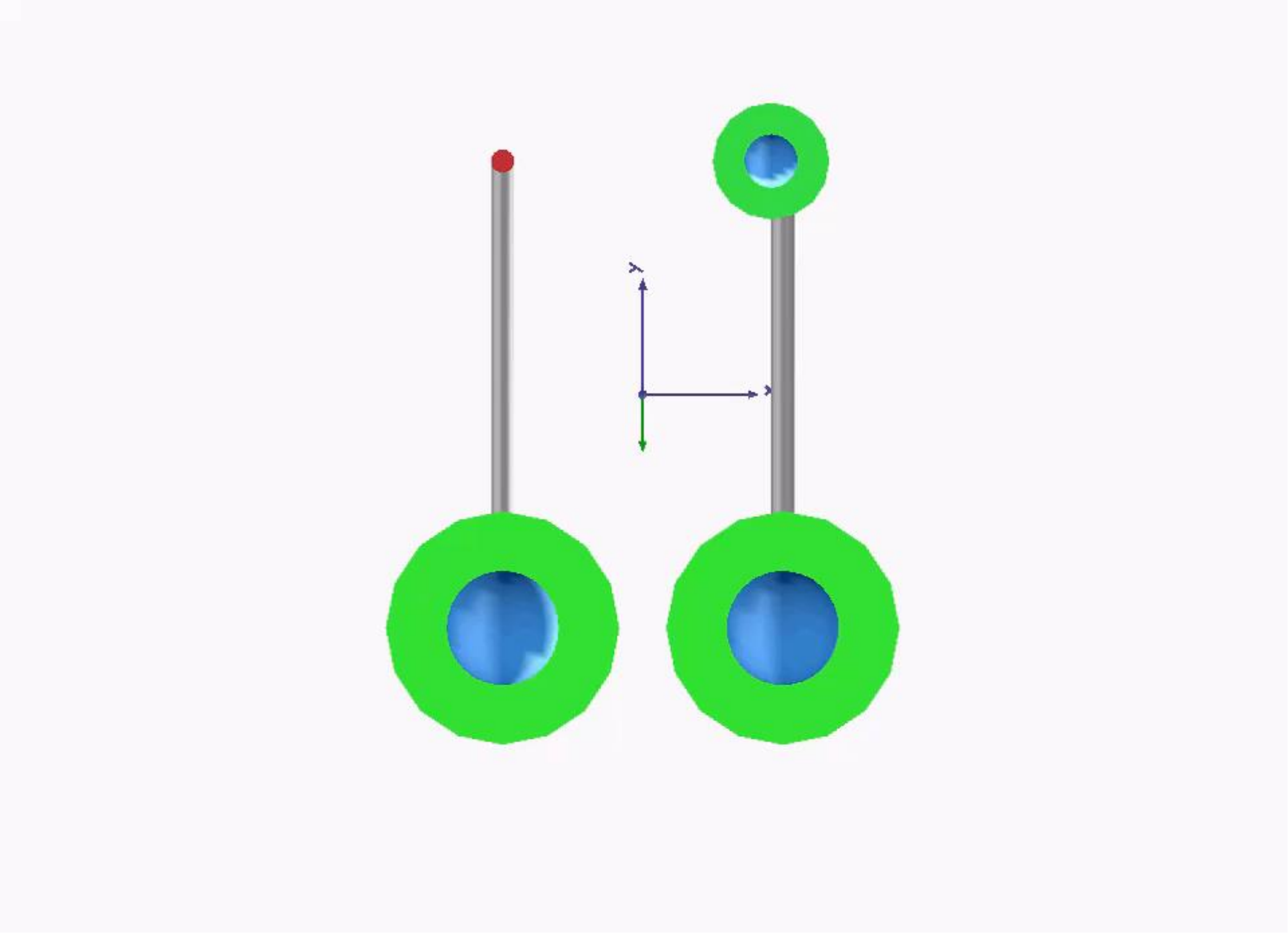
The implicit system, is a linear system of dimension 8. Easy to solve.

Despite the hard-impulses, we can use explicit integration method. This system was simulated with RKFIX3 at only 100Hz

Perfectly suited for hard real-time simulation (this was 300x too fast)

No need for complicated tooling to handle structural changes anymore….

Reformulation of multibody dynamics



More fun with hard contacts…

(again using explicit integration methods with fixed step size)

How about mechanics

Mass-Spring Approach
- No non-linear equation systems
- Many states
- Stiffness
- High-Frequency Oscillations

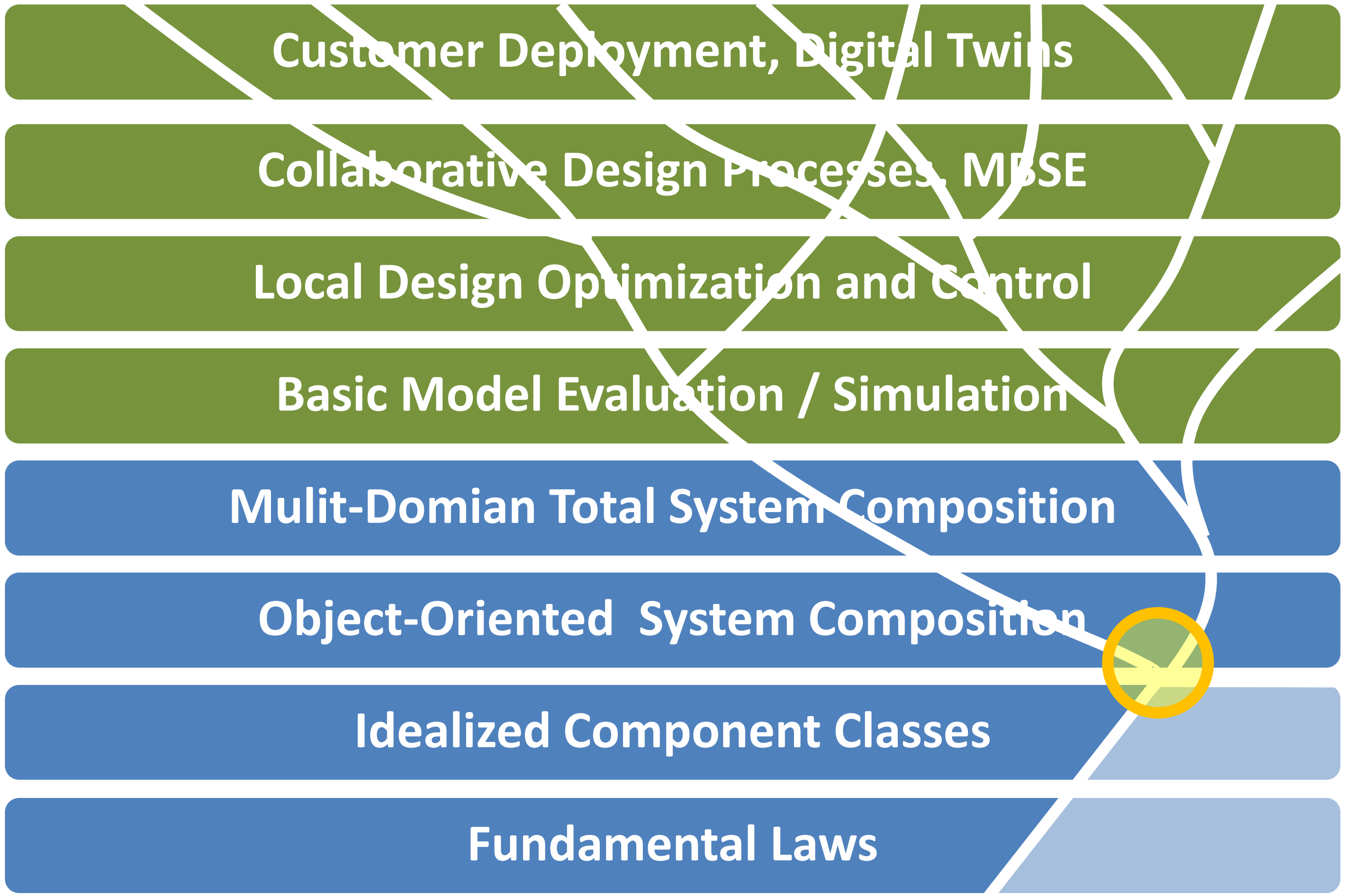
Dialectic Mechanics

Rigid Multibody
- Few states
- Very efficient
- Non-linear equation systems
- Not feasible for contacts

Robustnes

Performance

Let us tackle the problem at its root…



Take home lessons I

In the Modelica Association (but also in the M&S community)
we have misattributed our efforts:

- **Too much focus on languages, tools, and computation**

- **Too little focus on modelling, underlying principles and information**

There is a great potential to drastically reduce complexity
in the simulation of classic physical systems but you have
to deviate from classic textbook idealization

Take home lessons II

Find the DLR ThermoFluid Stream Library on GitHub:
https://github.com/DLR-SR/ThermofluidStream

Feel free to contribute!!

Release	**Dymola** Version 2021 (64-bit) 2020-04-17	**OpenModelica** OpenModelica v1.19.0-dev-38-ge9f86ba1ce (64-bit) OMSimulator v2.1.1.post80-g1bf17f4-mingw	**Modelon Impact**
v0.1-beta	(runs and passes regression test)		
v0.2-beta	(runs and passes regression test)	Fully compiles, mostly runs and passes regression test	(runs)
Latest main	(runs and passes regression test)	Fully compiles, mostly runs and passes regression test	(should run)

Papers on dialectic mechanics will be coming soon…
(and probably some sort of free library)