Transient Simulation of An Air-source Heat Pump under Cycling of Frosting and Reverse-cycle defrosting

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Outlines

• Introduction
  ▪ Motivation
  ▪ Objectives

• Development of a Dynamic Modeling Framework
  ▪ Reversible heat exchanger model
  ▪ Component models

• Model Validations
  ▪ Experimental facility
  ▪ Simulation results

• Conclusions
Motivation

- Cycling operations of frosting and reverse-cycle defrosting (RCD) is common for air-source heat pump (ASHP) units in winter operations.
- Significant computational complexities are involved in RCD modeling due to reverse refrigerant flow and coupled dynamics with frost.
Motivation

- Absence of a general simulation tool for capturing ASHP dynamics under frosting-defrosting cycling operations.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Operating Mode</th>
<th>Reverse flow</th>
<th>Experimental validation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Qiao et al. (2017)[1]</td>
<td>Heating</td>
<td>Not considered</td>
<td>Vapor injection heat pump</td>
</tr>
<tr>
<td>Steiner &amp; Riberer (2013)[2]</td>
<td>defrosting</td>
<td>Not considered</td>
<td>CO2 heat pump</td>
</tr>
<tr>
<td>Han et al. (2022)[3]</td>
<td>defrosting</td>
<td>Not considered</td>
<td>Residential R410A heat pump</td>
</tr>
<tr>
<td>Qiao et al. (2018)[4]</td>
<td>defrosting</td>
<td>Modeled</td>
<td>None</td>
</tr>
</tbody>
</table>

Literature review on system-level frosting/defrosting transient simulation
Objectives

• Develop a dynamic modeling framework for ASHP under cycling of frosting and reverse-cycle defrosting.

• Experimentally validate the model at heat pump cycle level.

• Improve model robustness for reversible ASHP systems incorporating frost formation and melting.
Reversible Heat Exchanger (HX) Model

Staggered grid for discretization

\[
\begin{bmatrix}
\dot{m}_1 \\
H_1
\end{bmatrix}
\begin{array}{cccc}
(p_1, h_1) \\
\bullet \\
\bullet
\end{array}
\begin{array}{c}
(p_i, h_i)
\bullet
\bullet
\bullet
\end{array}
\begin{array}{c}
(p_n, h_n)
\bullet
\bullet
\bullet
\end{array}
\begin{bmatrix}
\dot{m}_{n+1} \\
H_{n+1}
\end{bmatrix}
\]

Mass balance:

\[
V_i \left( \left( \frac{\partial \rho}{\partial h} \right)_i \frac{dp_i}{dt} + \left( \frac{\partial \rho}{\partial p} \right)_i \frac{dh_i}{dt} \right) = \dot{m}_i - \dot{m}_{i+1}
\]

Momentum balance:

\[
L_i \frac{d\dot{m}_i}{dt} = \rho_{i-1} \bar{v}_i^2 A_{i-1} - \rho_i \bar{v}_{i+1}^2 A_i + \frac{A_{i-1} + A_i}{2} (p_{i-1} - p_i) - F_{f,i}
\]

Energy balance:

\[
V_i \left( \left( \frac{h_i}{\partial h} \right)_i - 1 \right) \frac{dp_i}{dt} + \left( h_i \frac{\partial \rho}{\partial p}_i + \rho \right) \frac{dh_i}{dt} = H_i - H_{i+1} + \dot{Q}_i
\]

\[
F_f = A_s \Delta p_f \text{sign}(\dot{m})
\]

\[
\Delta p_f = f \frac{L \rho v^2}{D/2}
\]
HX Model: Metal Structure and Air-side

Metal wall energy balance:

\[ (M_{f\text{in}} c_{p,f\text{in}} + M_{t} c_{p,t}) \frac{dT_{w}}{dt} = \dot{Q}_{\text{in}} + \dot{Q}_{\text{out}} \]

Air side heat and mass transfer (forced convection):

\[ T_{a,\text{out}} = T_{as} + (T_{a,\text{in}} - T_{as}) e^{-Ntu} \]

\[ Ntu = \frac{\alpha_{h}(A_{t} + \eta_{f\text{in}}A_{f\text{in}})}{\dot{m} c_{p,a}} \]

\[ \omega_{a,\text{out}} = \omega_{a,\text{in}} - \left( 1 - e^{-\frac{Ntu}{Le^{2/3}}} \right) \max\{0, \omega_{a,\text{in}} - \omega_{as,s}\} \]
1-D Frost Formation

Mass conservation over frost layer:
\[
\frac{d(\rho_f \delta_f)}{dt} = \rho_f \frac{d\delta_f}{dt} + \delta_f \frac{d\rho_f}{dt} = \dot{m}_a''
\]

Mass and heat diffusions within frost layer:
\[
D_v \frac{\partial^2 \rho_v}{\partial x^2} = \xi \rho_v^{[1]}
\]
\[
k_f \frac{\partial^2 T_f}{\partial x^2} = -\Delta h_{sg} \xi \rho_v(x)
\]

Prescribed water vapor density BC
\[
\rho_f,\text{ref}[t + \Delta t] = \rho_f,\text{ref}[t] + \frac{\dot{m}_\rho''}{\delta_f,\text{ref}[t]} \Delta t
\]
first-order filter
\[
\frac{d\rho_f}{dt} = \frac{1}{\tau} \left( \rho_f,\text{ref} - \rho_f \right)
\]
\[
\frac{d\delta_f}{dt} = \frac{1}{\tau} \left( \delta_f,\text{ref} - \delta_f \right)
\]

quasi-steady-state frost growth
(update after a fixed time interval)
\[
\delta_f,\text{ref}[t + \Delta t] = \delta_f,\text{ref}[t] + \frac{\dot{m}_\delta''}{\delta_\rho,\text{ref}[t]} \Delta t
\]
1-D Frost Melting

Frost melting process progresses through predictable stages (5-stage applied here)[4]
Multi-stage Frost Melting Model

Dynamics of frost layer, water film, air gap of $i^{th}$ stage: $\dot{x} = f_i(x)$

\[ x = [T_f \quad \delta_f \quad T_{water} \quad \delta_{water} \quad T_{air} \quad \delta_{air} \quad \rho_f]^T \]

Switched dynamics between stages:

**Rule 1:** IF $T_w < 273.15$ K THEN $\dot{x} = f_1(x)$

⋮

**Rule 5:** IF $(T_w > 273.15$ K AND $\delta_{water} < \delta_{\text{min}}$ AND $\delta_f < \delta_{\text{min}}$) THEN $\dot{x} = f_5(x)$

Developed a robust switching algorithm based on the **Fuzzy Modeling** approach

\[ \dot{x} = \frac{\sum_{i=1}^{5} \omega_i f_i(x)}{\sum_{i=1}^{5} \omega_i} \]

weights $\omega_i(x, \mu)$
Incorporating Frost Models into HX Model

Goal: run frost formation and melting models simultaneously and switch dynamics

Finite-volume HX model incorporating frost layer

\[
\frac{d\rho_f}{dt} = \phi \left( \frac{1}{\tau} (\rho_{ref} - \rho_f) \right) + (1 - \phi) \left( f_{\rho,\text{melt}}(\rho_f, \delta_f) \right)
\]

Overall frost dynamics:

\[
\frac{d\delta_f}{dt} = \phi \left( \frac{1}{\tau} (\delta_{ref} - \delta_f) \right) + (1 - \phi) \left( f_{\delta,\text{melt}}(\rho_f, \delta_f) \right)
\]
Component Models

\[ \dot{m} = \rho_{in} \eta_v V_s \frac{N}{60} \]

\[ \dot{m} = C_d A_v \sqrt{2 \rho_{in} (p_{in} - p_{out})} \]

\[ \Delta p_{fan} = a_0 + a_1 \dot{V} + a_2 \dot{V}^2 + a_3 \dot{V}^3 \]

\[ \Delta p_i = \Delta p_{fan} \]

\[ \Sigma \dot{V}_i = \dot{V} \]

Solve for air flow rate distribution due to non-uniform frost formation.
## Component Models

### Accumulator

**Assumptions:**
1. Ideal phase separation
2. Vapor and liquid inside accumulator are saturated\[5\]

**Energy conservation:**
\[
\frac{d}{dt} \left( V_g \rho_g + V_f \rho_f \right) = \dot{m}_{in} - \dot{m}_{out}
\]

**Mass conservation:**
\[
\frac{d}{dt} \left( V_g \rho_g + V_f \rho_f \right) = \dot{m}_{in} h_{in} - \dot{m}_{out} h_g
\]

**Reversing valve**\[6\]

**Check valve model**
\[
\dot{m} = \phi A_v \sqrt{\rho_{in} \Delta p}
\]

\[
\dot{m} (h_{in} - h_{out}) = C_{HD} (T_{dis} - T_{suc})
\]

Heat transfer loss coefficient

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\[ \text{CHPB} \quad \text{CENTER FOR HIGH PERFORMANCE BUILDINGS AT PURDUE} \]

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A 2-ton commercially available residential heat pump
## Cycling of Frosting-Defrosting Operations

<table>
<thead>
<tr>
<th>Indoor</th>
<th>Indoor</th>
<th>Outdoor</th>
<th>Outdoor</th>
</tr>
</thead>
<tbody>
<tr>
<td>dry-bulb [°F]</td>
<td>RH [%]</td>
<td>dry-bulb [°F]</td>
<td>RH [%]</td>
</tr>
<tr>
<td>65</td>
<td>40%</td>
<td>28</td>
<td>85%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time [min]</th>
<th>Frequency [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>80</td>
</tr>
<tr>
<td>20</td>
<td>80</td>
</tr>
<tr>
<td>40</td>
<td>80</td>
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<td>60</td>
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</tr>
<tr>
<td>160</td>
<td>80</td>
</tr>
<tr>
<td>180</td>
<td>80</td>
</tr>
</tbody>
</table>

- **Defrosting**
- **Heating**

**Compressor speed**
Simulation Results: refrigerant pressures

![Graph showing refrigerant pressures over time with heating and defrosting phases indicated.](image-url)
Simulation Results

Refrigerant liquid-line mass flow rate

Compressor power & Indoor air-side capacity
Simulation Results: frost dynamics

23% of air flow passage was blocked after 2-hour frosting operation
Time Evolution of Non-uniform Frost Formation

Frost density

Frost thickness

Air flow rate
Defrost Efficiency

Heat supply to melt frost and vaporize retained water:

\[ Q_{\text{melt}} = M_f (\Delta h_{sf} + c_p (T_{\text{melt}} - T_f)) \quad Q_{\text{vaporize}} = M_{\text{retained}} \Delta h_{fg} \]

\[ \eta_d = \frac{Q_{\text{melt}} + Q_{\text{vaporize}}}{\int (\dot{W}_\text{comp} + \dot{W}_\text{fan} + \dot{Q}_\text{ID}) \, dt} \]

<table>
<thead>
<tr>
<th>Defrost cycles</th>
<th>1(^{\text{st}}) cycle</th>
<th>2(^{\text{nd}}) cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta_d )</td>
<td>30%</td>
<td>56%</td>
</tr>
</tbody>
</table>
Conclusions

• Developed a comprehensive dynamic modeling framework for air-source heat pumps under cycling of frosting and reverse-cycle defrosting.

• Conducted experimental tests of a residential heat pump unit for model validations.

• The developed model can predict system transients under cycling operations with satisfactory accuracy and computational speed (RTF 0.49).
References


Questions?

Funding:
Appendix

Fan model implementing a robust formulation for modeling air flow maldistribution\cite{7}

\[ \Delta p_{\text{rise}} = a_0 + a_1 \dot{V} + a_2 \dot{V}^2 + a_3 \dot{V}^3 \]

\[ \Delta p_1 = \Delta p_{\text{rise}} \]
\[ \Delta p_2 = \Delta p_{\text{rise}} \]
\[ \Delta p_N = \Delta p_{\text{rise}} \]
\[ \sum \dot{V}_i = \dot{V} \]

A large nonlinear algebraic equation system!
Appendix

Denote inertial pressure $r$

\[-\tau \frac{dm}{dt} = \Delta r\]

Define steady mass flow pressure $\hat{p}$

$p = r + \hat{p}$

Momentum balance

\[\Delta \hat{p} \approx \rho \nu \Delta \nu(\hat{p}, \dot{m}) - \Delta p_{ext}(\hat{p}, \dot{m})\]

downstream pressure can be computed explicitly

upstream is known

A new fan model