Transient Simulation of An Air-source Heat Pump under Cycling of Frosting and Reverse-cycle defrosting

Jiacheng Ma[1], Donghun Kim[2], James E. Braun[1]

[1]Ray W. Herrick Laboratories, School of Mechanical Engineering, Purdue University
West Lafayette, IN, U.S.
[2]Building Technology & Urban Systems Division, Lawrence Berkeley National Laboratory
Berkeley, CA, U.S.

PURDUE UNIVERSITY The Ray W. Herrick Laboratories

October 8, 2022

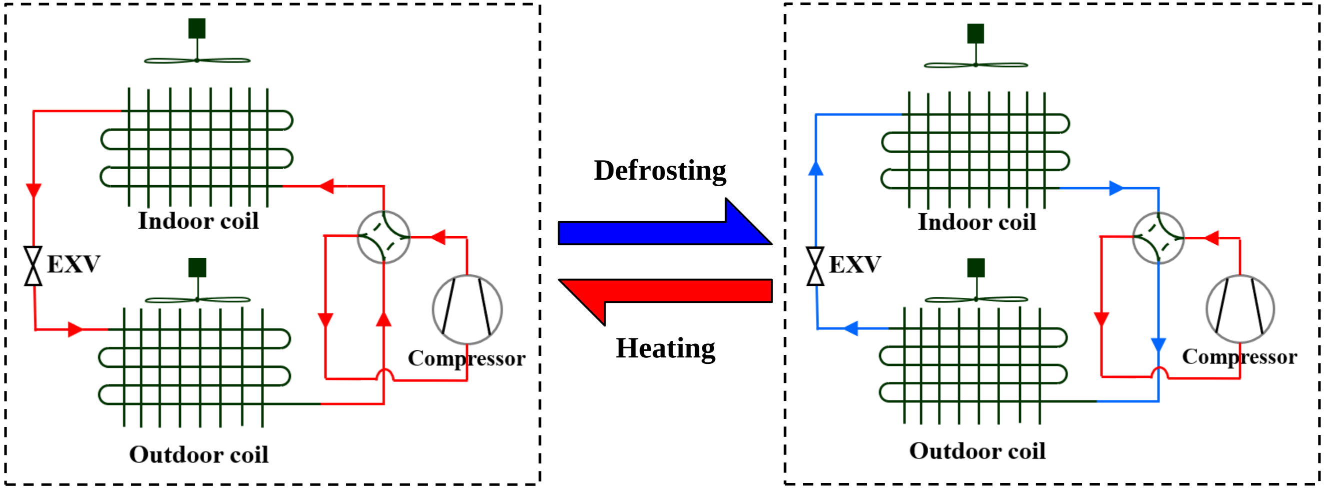
CHPB CENTER FOR HIGH PERFORMANCE BUILDINGS AT PURDUE

Outlines

- Introduction
  - Motivation
  - Objectives
- Development of a Dynamic Modeling Framework
  - Reversible heat exchanger model
  - Component models
- Model Validations
  - Experimental facility
  - Simulation results
- Conclusions

Motivation

- Cycling operations of frosting and reverse-cycle defrosting (RCD) is common for air-source heat pump (ASHP) units in winter operations.
- Significant computational complexities are involved in RCD modeling due to reverse refrigerant flow and coupled dynamics with frost.



Motivation

- ❑ Absence of a general simulation tool for capturing ASHP dynamics under frosting-defrosting cycling operations.

Reference	Operating Mode	Reverse flow	Experimental validation
Qiao et al. (2017)[1]	Heating	Not considered	Vapor injection heat pump
Steiner & Riberer (2013)[2]	defrosting	Not considered	CO2 heat pump
Han et al. (2022)[3]	defrosting	Not considered	Residential R410Aheat pump
Qiao et al. (2018)[4]	defrosting	Modeled	None

Literature review on **system-level** frosting/defrosting transient simulation

Objectives

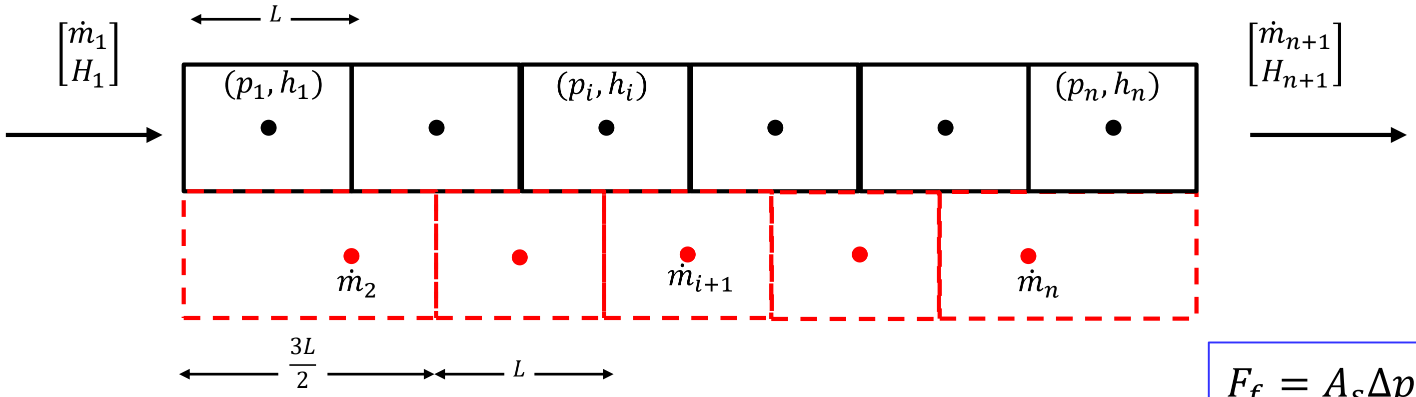
- Develop a dynamic modeling framework for ASHP under cycling of frosting and reverse-cycle defrosting.

- Experimentally validate the model at heat pump cycle level.

- Improve model robustness for reversible ASHP systems incorporating frost formation and melting.

Reversible Heat Exchanger (HX) Model

Staggered grid for discretization



Mass balance:

$$V_i\left[\left(\frac{\partial \rho}{\partial p}\bigg|_h\right)_i \frac{dp_i}{dt} + \left(\frac{\partial \rho}{\partial h}\bigg|_p\right)_i \frac{dh_i}{dt}\right] = \dot m_i - \dot m_{i+1}$$

Momentum balance:

$$L_i \frac{d\dot m_i}{dt} = \rho_{i-1}\bar v_i^2 A_{i-1} - \rho_i \bar v_{i+1}^2 A_i + \frac{A_{i-1} + A_i}{2}(p_{i-1} - p_i) - F_{f,i}$$

Energy balance:

$$V_i\left[\left(h_i \frac{\partial \rho}{\partial p}\bigg|_h - 1\right)_i \frac{dp_i}{dt} + \left(h_i \frac{\partial \rho}{\partial h}\bigg|_p + \rho\right)_i \frac{dh_i}{dt}\right] = H_i - H_{i+1} + \dot Q_i$$

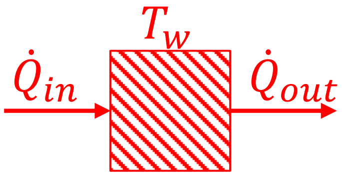
$$F_f = A_s \Delta p_f \, sign(\dot m)$$

$$\Delta p_f = f \frac{L}{D}\frac{\rho v^2}{2}$$

HX Model: Metal Structure and Air-side

Metal wall energy balance:

$$\left(M_{fin} c_{p,fin} + M_t c_{p,t}\right) \frac{dT_w}{dt} = \dot{Q}_{in} + \dot{Q}_{out}$$

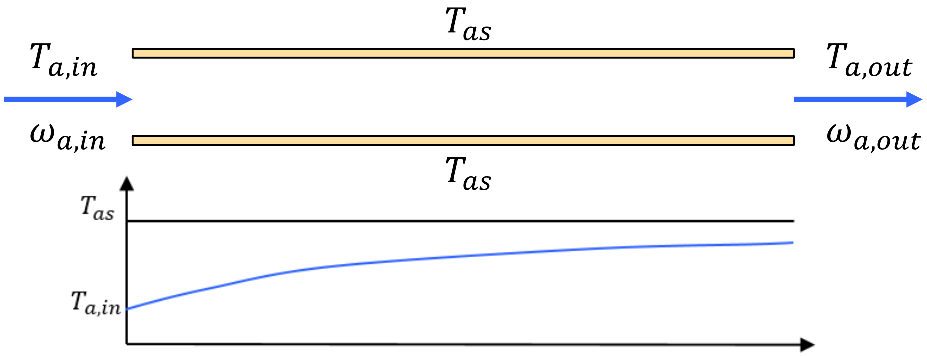


Air side heat and mass transfer (forced convection):

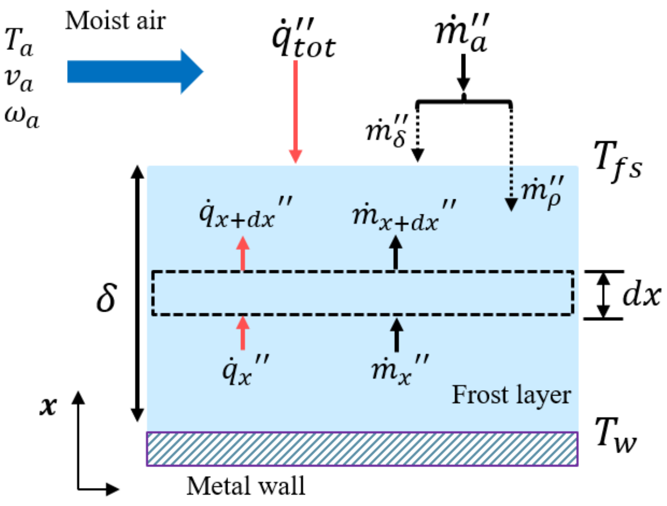
$$T_{a,out} = T_{as} + \left(T_{a,in} - T_{as}\right) e^{-Ntu}$$

$$Ntu = \frac{\alpha_h\left(A_t + \eta_{fin} A_{fin}\right)}{\dot{m} c_{p,a}}$$

$$\omega_{a,out} = \omega_{a,in} - \left(1 - e^{-\frac{Ntu}{Le^{2/3}}}\right) max\{0, \omega_{a,in} - \omega_{as,s}\}$$



1-D Frost Formation



Mass conservation over frost layer

$$\frac{d(\rho_f\delta_f)}{dt} = \rho_f \underset{\substack{\uparrow \\ \dot{m}''_\delta}}{\frac{d\delta_f}{dt}} + \delta_f \underset{\substack{\uparrow \\ \dot{m}''_\rho}}{\frac{d\rho_f}{dt}} = \dot{m}''_a$$

Mass and heat diffusions within frost layer:

$$D_v \frac{\partial^2 \rho_v}{\partial x^2} = \xi \rho_v^{[1]}$$

$$k_f \frac{\partial^2 T_f}{\partial x^2} = -\Delta h_{sg} \xi \rho_v(x)$$

Prescribed water vapor density BC

quasi-steady-state frost growth

(update after a fixed time interval)

$$\rho_{f,ref}[t+\Delta t] = \rho_{f,ref}[t] + \frac{\dot{m}''_\rho}{\delta_{f,ref}[t]}\Delta t$$

$$\delta_{f,ref}[t+\Delta t] = \delta_{f,ref}[t] + \frac{\dot{m}''_\delta}{\delta_{\rho,ref}[t]}\Delta t$$

first-order filter

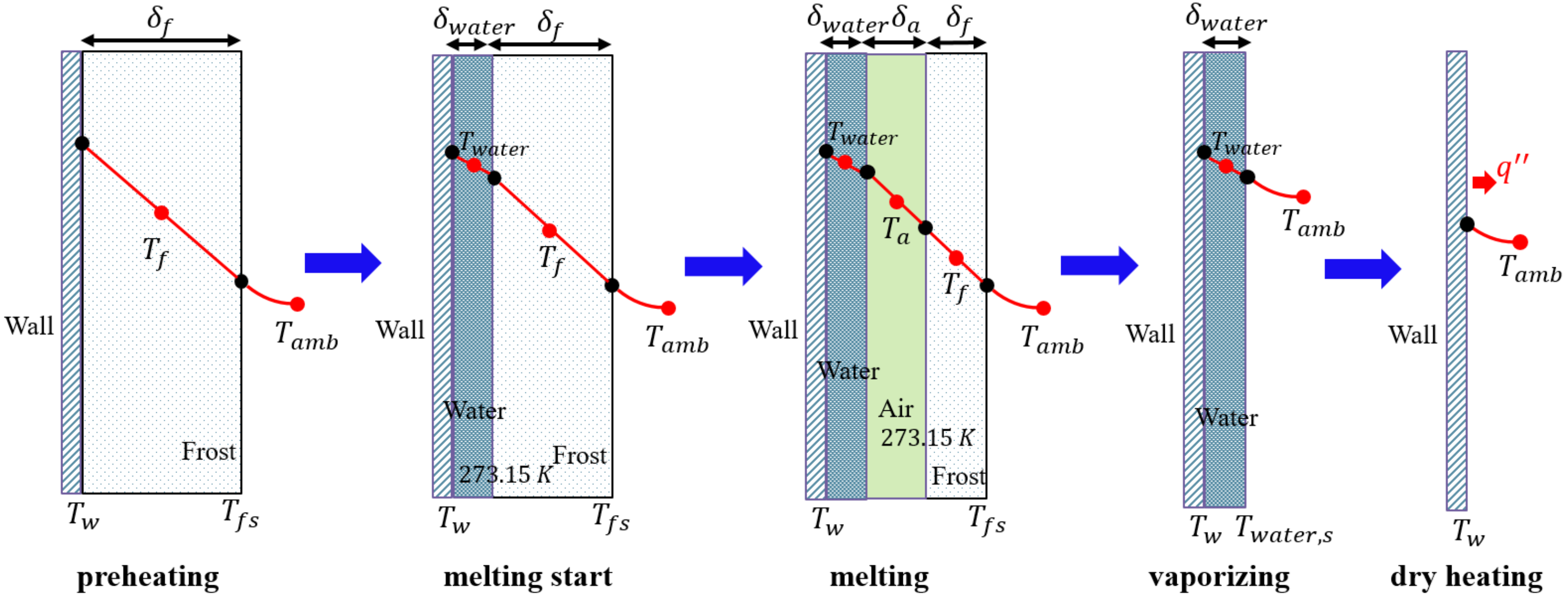
$$\frac{d\rho_f}{dt} = \frac{1}{\tau}(\rho_{f,ref} - \rho_f)$$

$$\frac{d\delta_f}{dt} = \frac{1}{\tau}(\delta_{f,ref} - \delta_f)$$

make $\boldsymbol{\delta_f}, \boldsymbol{\rho_f}$ state variables in CT

1-D Frost Melting

Frost melting process progresses through predictable stages (5-stage applied here)[4]



Multi-stage Frost Melting Model

Dynamics of frost layer, water film, air gap of i^{th} stage: $\dot{x} = f_i(x)$

$$x = [T_f \quad \delta_f \quad T_{water} \quad \delta_{water} \quad T_{air} \quad \delta_{air} \quad \rho_f\,]^T$$

Switched dynamics between stages:

Rule 1: **IF** $T_w < 273.15$ K **THEN** $\dot{x} = f_1(x)$

⋮

Rule 5: **IF** ($T_w > 273.15$ K AND $\delta_{water} < \delta_{min}$ AND $\delta_f < \delta_{min}$) **THEN** $\dot{x} = f_5(x)$

IF-THEN rules are discontinuous and numerically inefficient!

Developed a robust switching algorithm based on the **Fuzzy Modeling** approach

$$\dot{x} = \frac{\sum_{i=1}^{5} \omega_i \, f_i(x)}{\sum_{i=1}^{5} \omega_i}$$

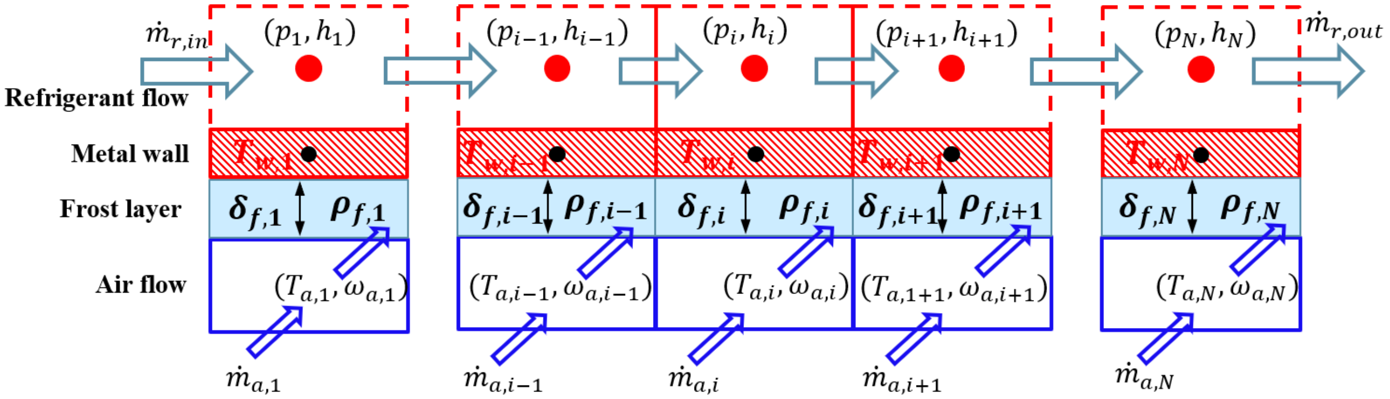
weights $\omega_i(x, \mu)$

PURDUE UNIVERSITY The Ray W. Herrick Laboratories

CHPB CENTER FOR HIGH PERFORMANCE BUILDINGS AT PURDUE

Incorporating Frost Models into HX Model

Goal: run frost formation and melting models simultaneously and switch dynamics



Finite-volume HX model incorporating frost layer

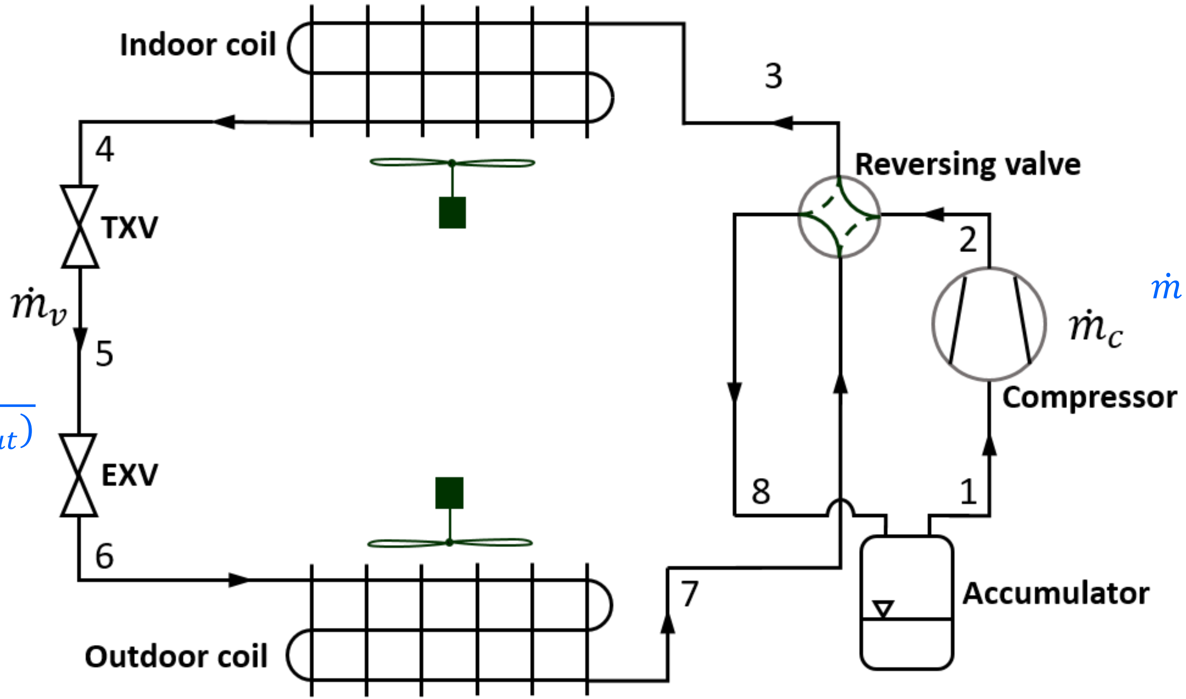
Overall frost dynamics:

$$\frac{d\rho_f}{dt} = \phi\left(\frac{1}{\tau}\left(\rho_{ref} - \rho_f\right)\right) + (1-\phi)\left(f_{\rho,melt}(\rho_f, \delta_f)\right)$$

$$\frac{d\delta_f}{dt} = \phi\left(\frac{1}{\tau}\left(\delta_{ref} - \delta_f\right)\right) + (1-\phi)\left(f_{\delta,melt}(\rho_f, \delta_f)\right)$$

formation *melting*

Component Models



$$\dot{m} = \rho_{in}\eta_v V_s \frac{N}{60}$$

$$\dot{m} = C_d A_v \sqrt{2\rho_{in}(p_{in} - p_{out})}$$

$$\Delta p_{fan} = a_0 + a_1\,\dot{V} + a_2\,\dot{V}^2 + a_3\,\dot{V}^3$$

$$\Delta p_i = \Delta p_{fan}$$

$$\sum \dot{V}_i = \dot{V}$$

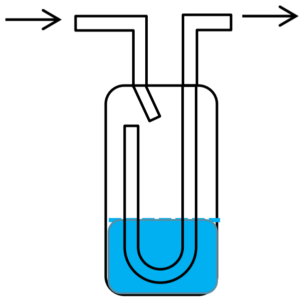
Solve for air flow rate distribution due to non-uniform frost formation

PURDUE UNIVERSITY | The Ray W. Herrick Laboratories

CHPB CENTER FOR HIGH PERFORMANCE BUILDINGS AT PURDUE

Component Models

Accumulator



Assumptions:

1. Ideal phase separation
2. Vapor and liquid inside accumulator are saturated[5]

Mass conservation: $$\frac{d}{dt}\left(V_g\rho_g + V_f\rho_f\right) = \dot{m}_{in} - \dot{m}_{out}$$

Energy conservation: $$\frac{d}{dt}\left(V_g\rho_g u_g + V_f\rho_f u_f\right) = \dot{m}_{in}h_{in} - \dot{m}_{out}h_g$$

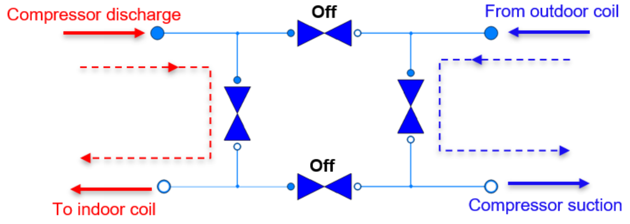
$$V_f + V_g = V_{acc}$$

Reversing valve[6]

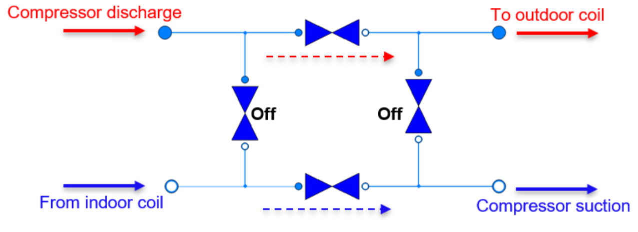
Check valve model $$\dot{m} = \phi A_v\sqrt{\rho_{in}\Delta p}$$

$$\dot{m}(h_{in} - h_{out}) = C_{HD}(T_{dis} - T_{suc})$$

Heat transfer loss coefficient

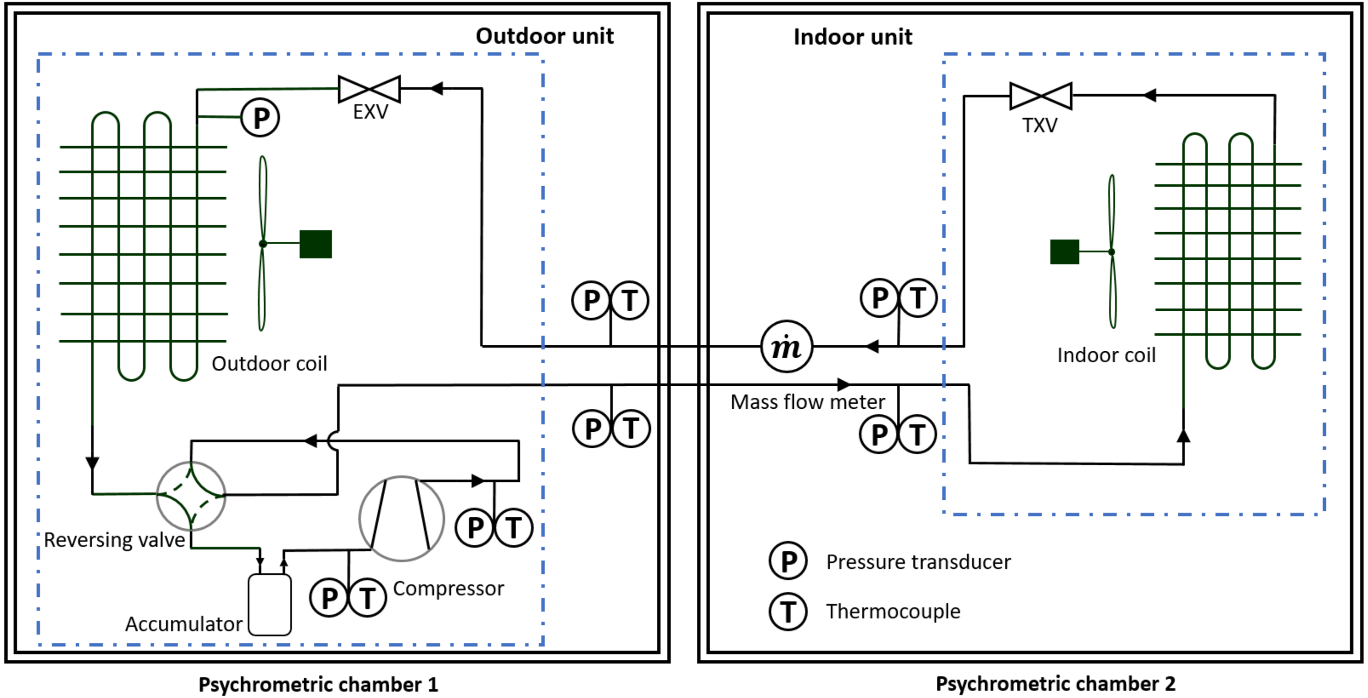


Heating mode



Cooling/Defrosting mode

Experimental Facility



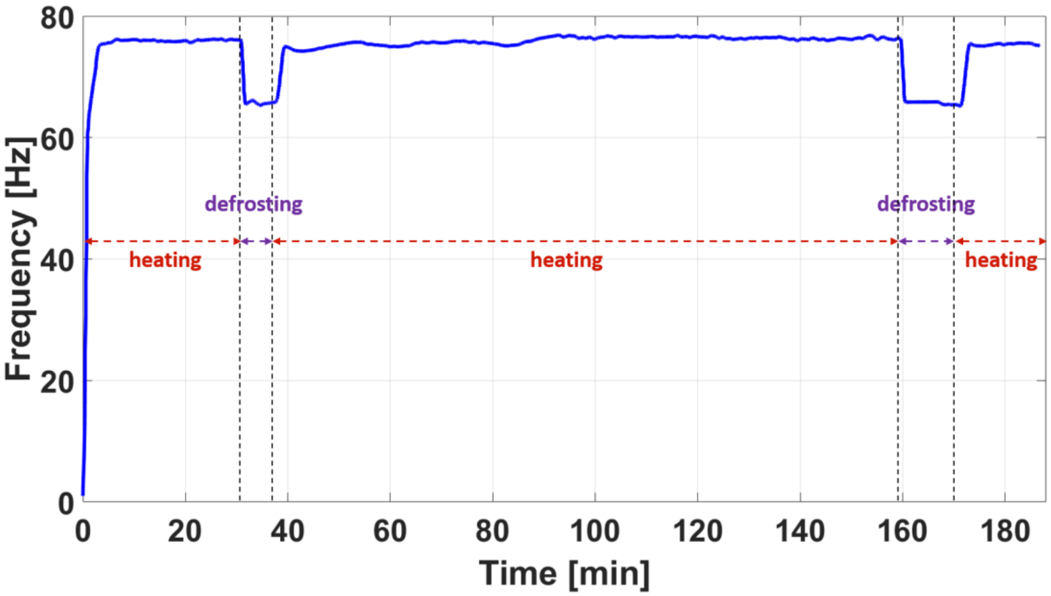
A 2-ton commercially available residential heat pump

PURDUE UNIVERSITY | The Ray W. Herrick Laboratories

CHPB CENTER FOR HIGH PERFORMANCE BUILDINGS AT PURDUE

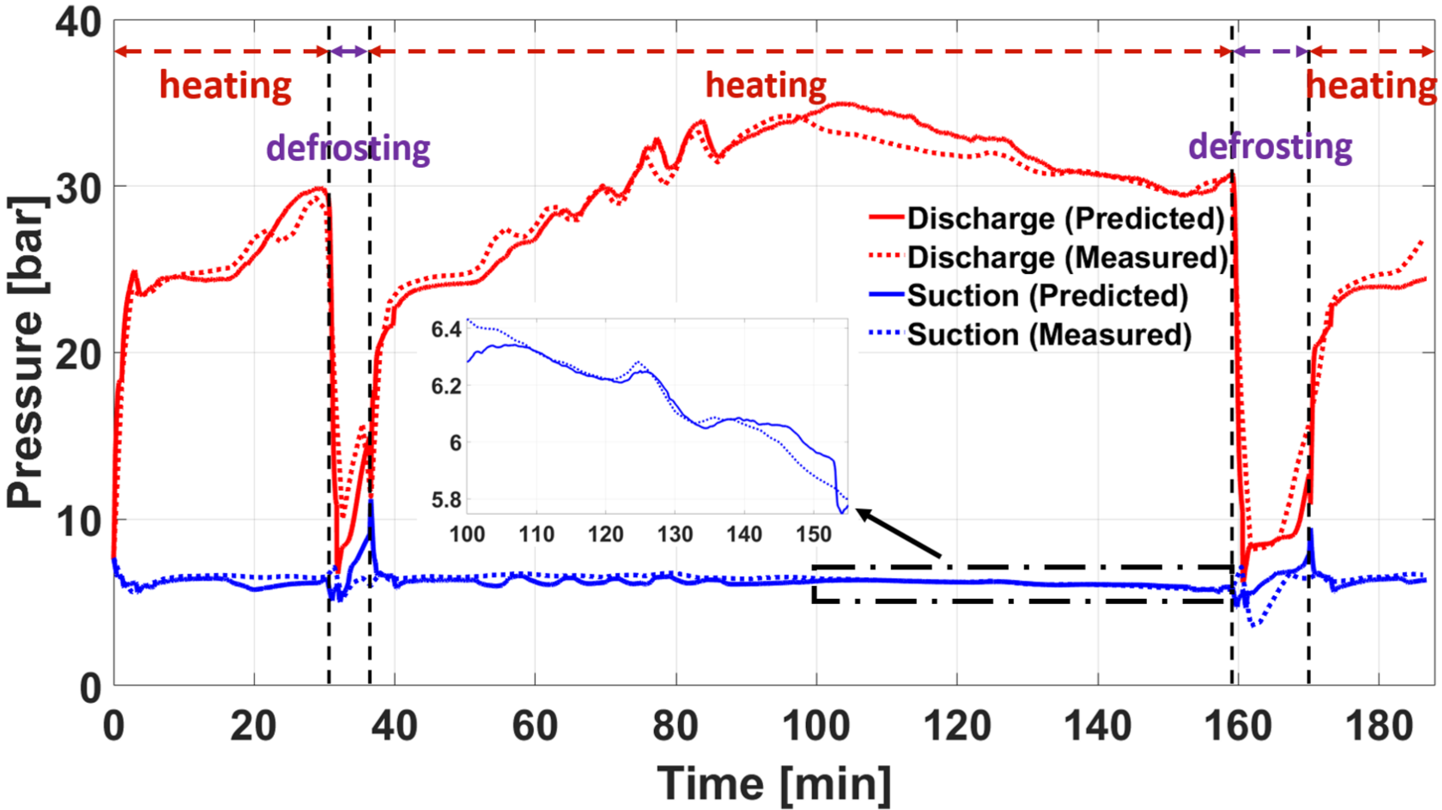
Cycling of Frosting-Defrosting Operations

Indoor dry-bulb [°F]	Indoor RH [–]	Outdoor dry-bulb [°F]	Outdoor RH [–]
65	40%	28	85%

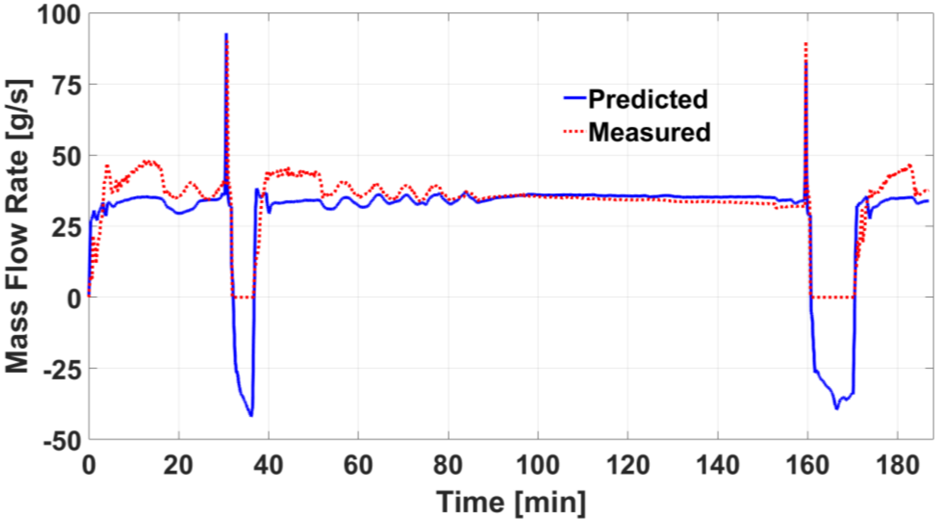


Compressor speed

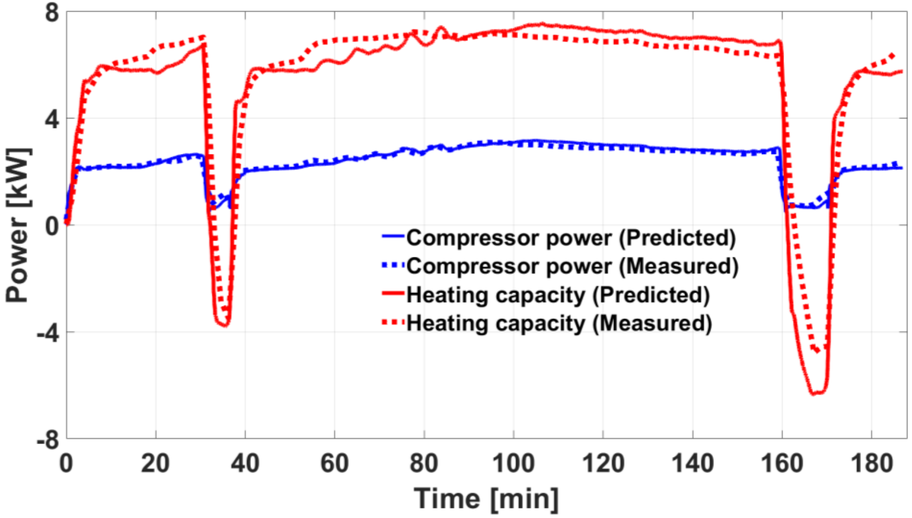
Simulation Results: refrigerant pressures



Simulation Results

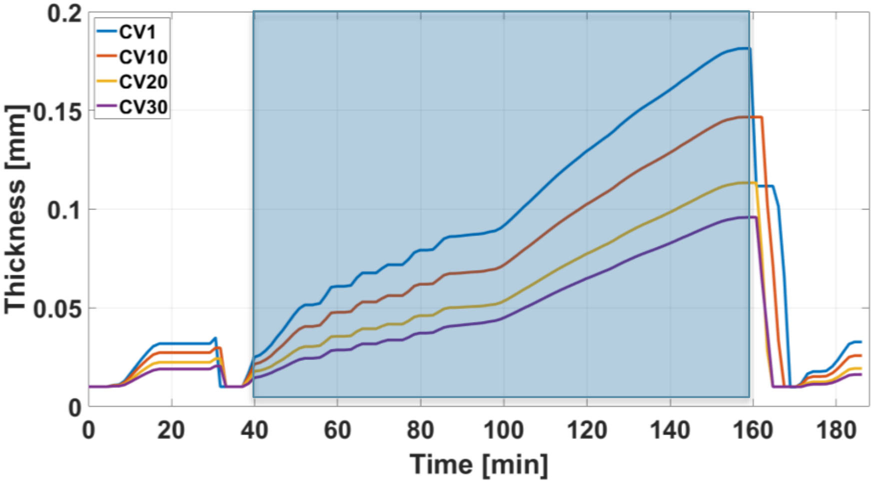


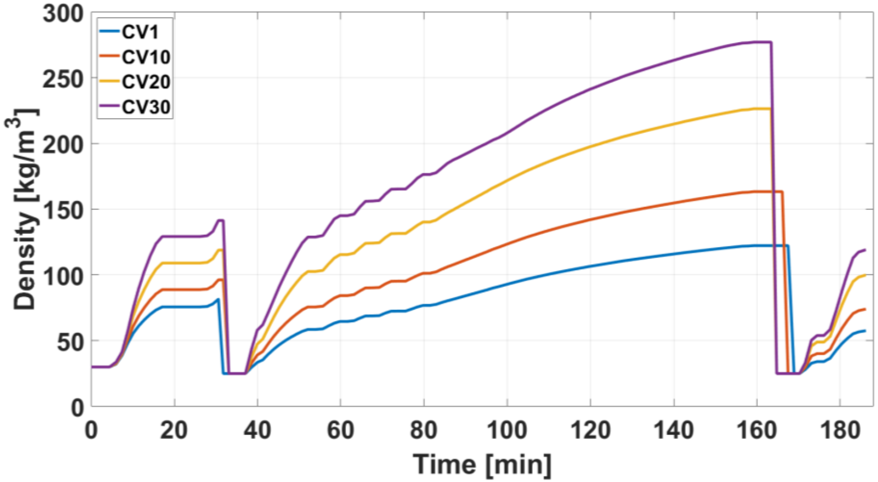
Refrigerant liquid-line mass flow rate



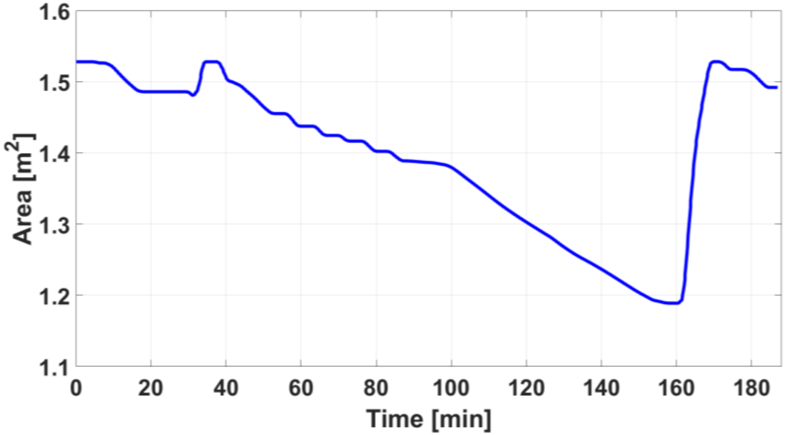
Compressor power & Indoor air-side capacity

Simulation Results: frost dynamics

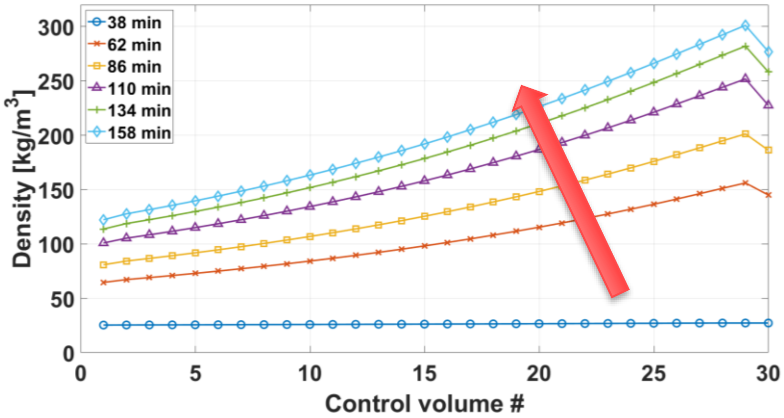




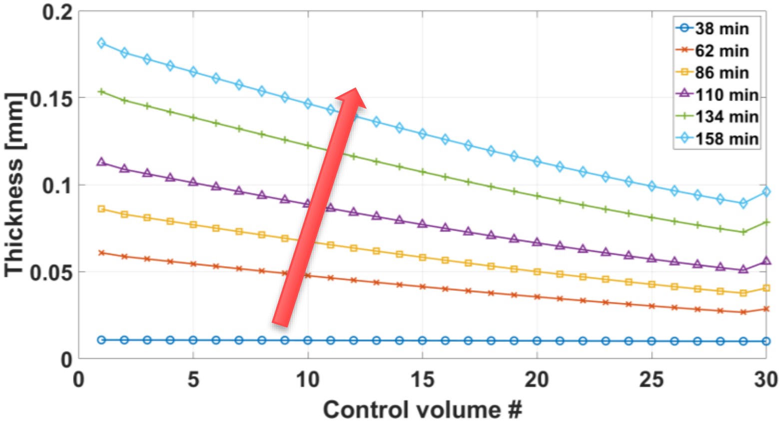
23% of air flow passage was blocked after 2-hour frosting operation



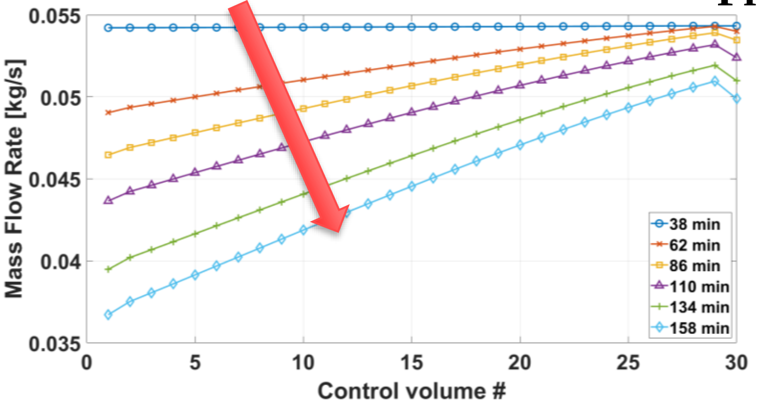
Time Evolution of Non-uniform Frost Formation



Frost density



Frost thickness

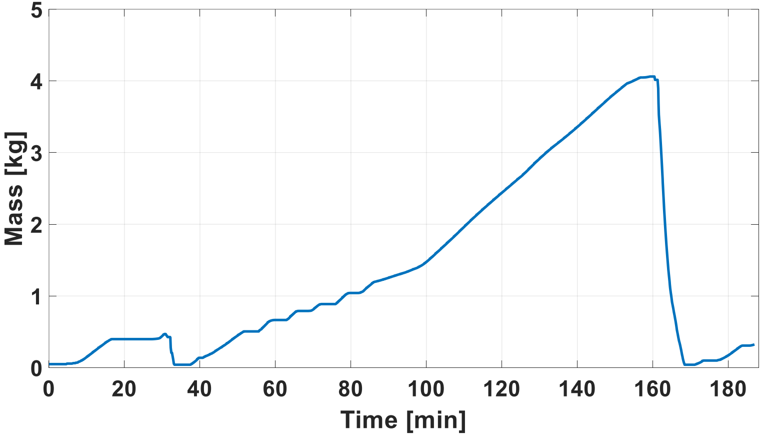


Air flow rate

Defrost Efficiency

Heat supply to melt frost and vaporize retained water:

$$Q_{melt} = M_f(\Delta h_{sf} + c_p\,(T_{melt} - T_f)) \qquad Q_{vaporize} = M_{retained}\Delta h_{fg}$$



$$\eta_d = \frac{Q_{melt} + Q_{vaporize}}{\int\left(\dot{W}_{comp} + \dot{W}_{fan} + \dot{Q}_{ID}\right)dt}$$

Compressor power Indoor fan power Indoor air-side capacity

Defrost cycles	1st cycle	2nd cycle
η_d	30%	56%

Conclusions

- Developed a comprehensive dynamic modeling framework for air-source heat pumps under cycling of frosting and reverse-cycle defrosting.

- Conducted experimental tests of a residential heat pump unit for model validations.

- The developed model can predict system transients under cycling operations with satisfactory accuracy and computational speed (RTF 0.49).

References

[1] Qiao, Hongtao, Vikrant Aute, and Reinhard Radermacher (2017). “Dynamic modeling and characteristic analysis of a two-stage vapor injection heat pump system under frosting conditions”. In: International Journal of Refrigeration 84, pp. 181–197.

[2] Steiner, Alois and René Rieberer (2013). “Parametric analysis of the defrosting process of a reversible heat pump system for electric vehicles”. In: Applied thermal engineering 61.2, pp. 393–400.

[3] Han, Binglong, Tong Xiong, Shijie Xu, Guoqiang Liu, and Gang Yan (2022). “Parametric study of a room air conditioner during defrosting cycle based on a modified defrosting model”. In: Energy 238, p. 121658.

[4] Qiao, Hongtao, Vikrant Aute, and Reinhard Radermacher (2018). “Modeling of transient characteristics of an air source heat pump with vapor injection during reverse-cycle defrosting”. In: International journal of Refrigeration 88, pp. 24–34.

[5] Pollock, Daniel T, Zehao Yang, John T Wen, Yoav Peles, and Michael K Jensen (2013). “Modeling and control of single and multiple evaporator vapor compression cycles for electronics cooling”. In: 2013 American Control Conference. IEEE, pp. 1645–1650.

[6] Qiao, Hongtao, Vikrant Aute, and Reinhard Radermacher (2015). “Transient modeling of a flash tank vapor injection heat pump system–Part I: Model development”. In: International journal of refrigeration 49, pp. 169–182.

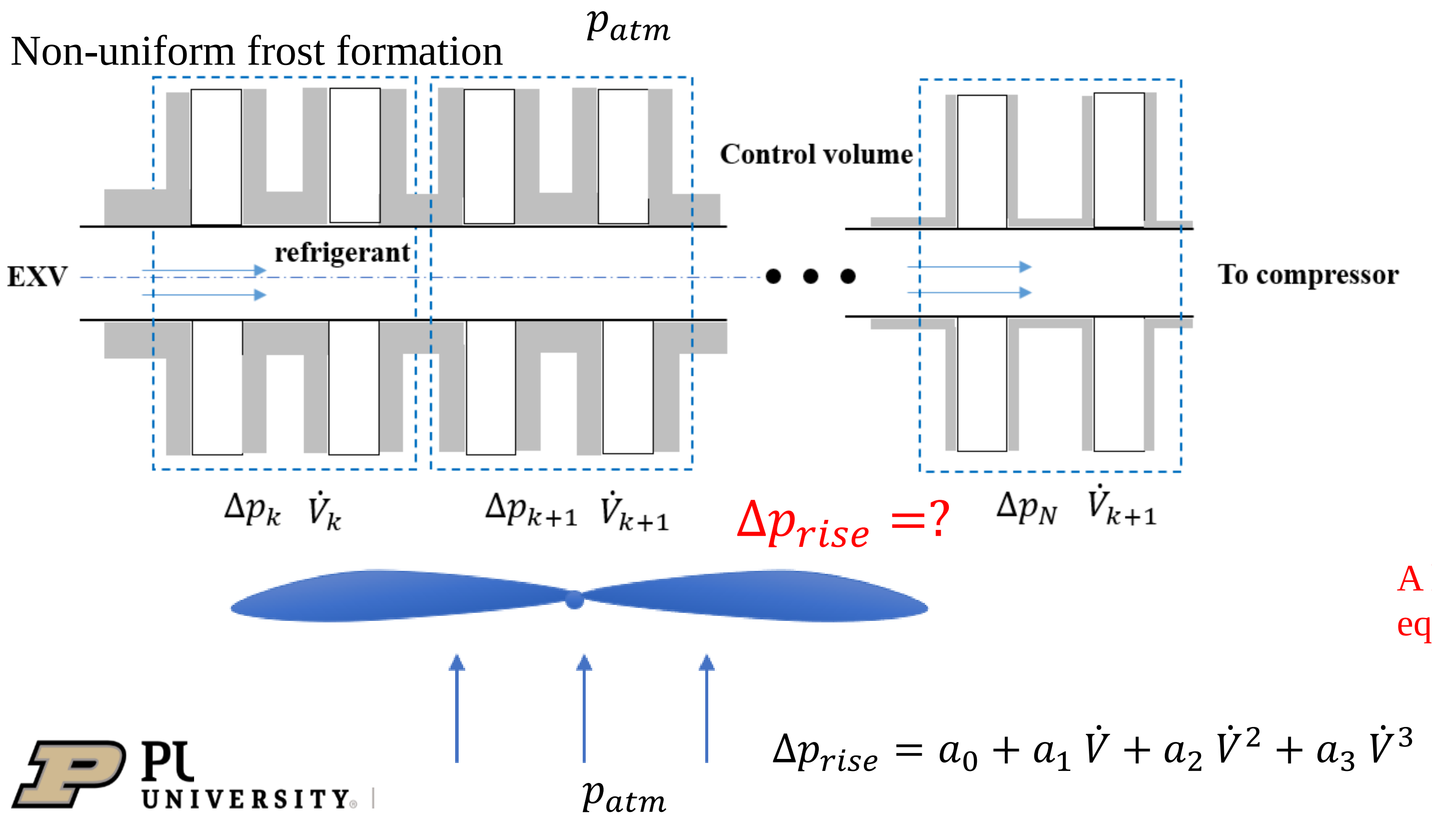
[7] Zimmer, Dirk (2020). “Robust object-oriented formulation of directed thermofluid stream networks”. In: Mathematical and Computer Modelling of Dynamical Systems 26.3, pp. 204–233.

Questions?

Funding: CHPB CENTER FOR HIGH PERFORMANCE BUILDINGS AT PURDUE ASHRAE®

Appendix

Fan model implementing a robust formulation for modeling air flow maldistribution[7]



$$\Delta p_1 = \Delta p_{rise}$$
$$\Delta p_2 = \Delta p_{rise}$$
$$\vdots$$
$$\Delta p_N = \Delta p_{rise}$$
$$\sum \dot{V}_i = \dot{V}$$

A large nonlinear algebraic equation system!

$$\Delta p_{rise} = a_0 + a_1\,\dot{V} + a_2\,\dot{V}^2 + a_3\,\dot{V}^3$$

Appendix

Denote inertial pressure *r*

$$-\tau \frac{d\dot{m}}{dt} = \Delta r$$

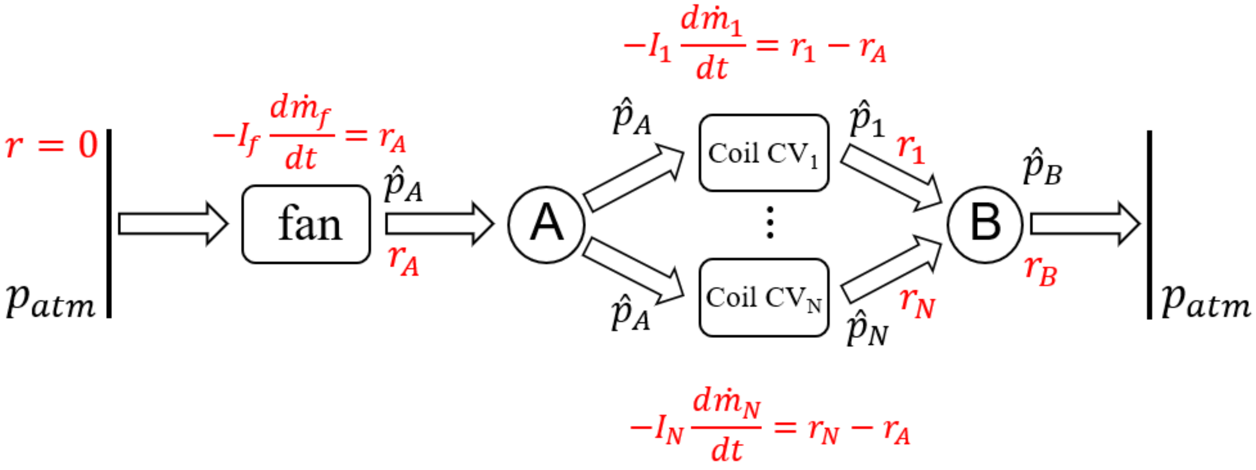
Define *steady mass flow pressure* $\hat{p}$

$$p = r + \hat{p}$$

Momentum balance

$$\Delta\hat{p} \cong \rho v \Delta v(\hat{p}, \dot{m}) - \Delta p_{ext}(\hat{p}, \dot{m})$$

downstream pressure can be computed explicitly upstream is known



A new fan model